CHAPTER # 2 CYLINDRICAL SYNCHRONOUS GENERATOR AND MOTOR

1. Introduction

Synchronous machines are named by this name as their speed is directly related to the line frequency. Synchronous machines may be operated either as motor or generator. Synchronous generators are called alternators. Synchronous motors are used mainly for power factor correction when operate at no load. Synchronous machines are usually constructed with stationary stator (armature) windings that carrying the current and rotating rotor (field) winding that create the magnetic flux. This flux is produced by DC current from a separate source (e.g. DC shunt generator).

2. Types of Synchronous Generators

The type of synchronous generators depends on the prime mover type as:

a) Steam turbines: synchronous generators that driven by steam turbine are high speed machines and known as turbo alternators. The maximum rotor speed is 3600 rpm corresponding 60 Hz and two poles.

b) Hydraulic turbines: synchronous generators that driven by water turbine are with speed varies from 50 to 500 rpm. The type of turbine to be used depends on the water head. For water head of 400m, Pelton wheel turbines are used. But for water head up to 350 m, Francis Turbines are used. For water head up to 50 m, Kaplan Turbines are used.

c) Diesel Engines: they are used as prime movers for synchronous generator of small ratings.

For the type a) and c) the rotor shape is cylindrical as shown in Fig. 1, and this type is called cylindrical rotor synchronous machines. But for type b), the rotor has salient poles on the rotor periphery as shown in Fig. 2.

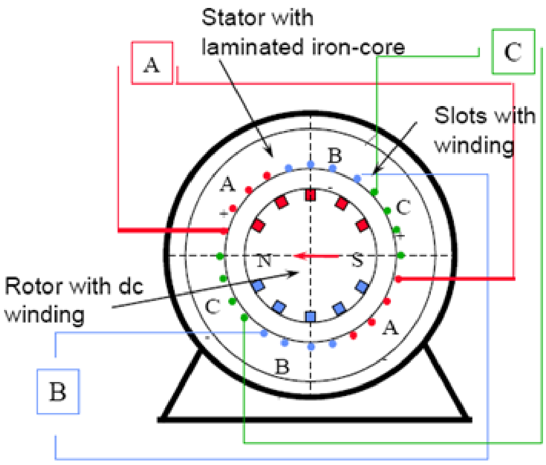


Fig.1, Cylindrical-type synchronous machine

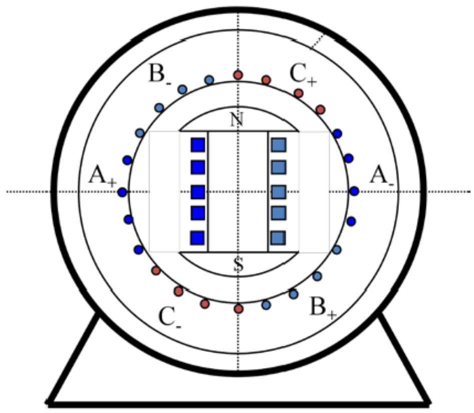


Fig.2, Salient-pole synchronous machine

3. Relation Between Field Flux and Armature Flux

There are two types of fluxes are obtained in the air gap between the armature and rotor;

- Field flux (F_f) that produced by the DC exciter and by using the appropriate prime mover, it can rotate at synchronous speed (N_s).

- Armature flux or armature reaction (F_a) which is produced from 3-phase armature winding and rotate with the synchronous speed.

Therefore, both of F_f and F_a are stationary w.r.t. each other

4. Equivalent Electrical Circuit Model per Phase

Neglecting saturation, the circuit diagrams, shown in Fig. 3, illustrate the per phase equivalent circuits of a cylindrical rotor synchronous machine in the motor and generator mode respectively.

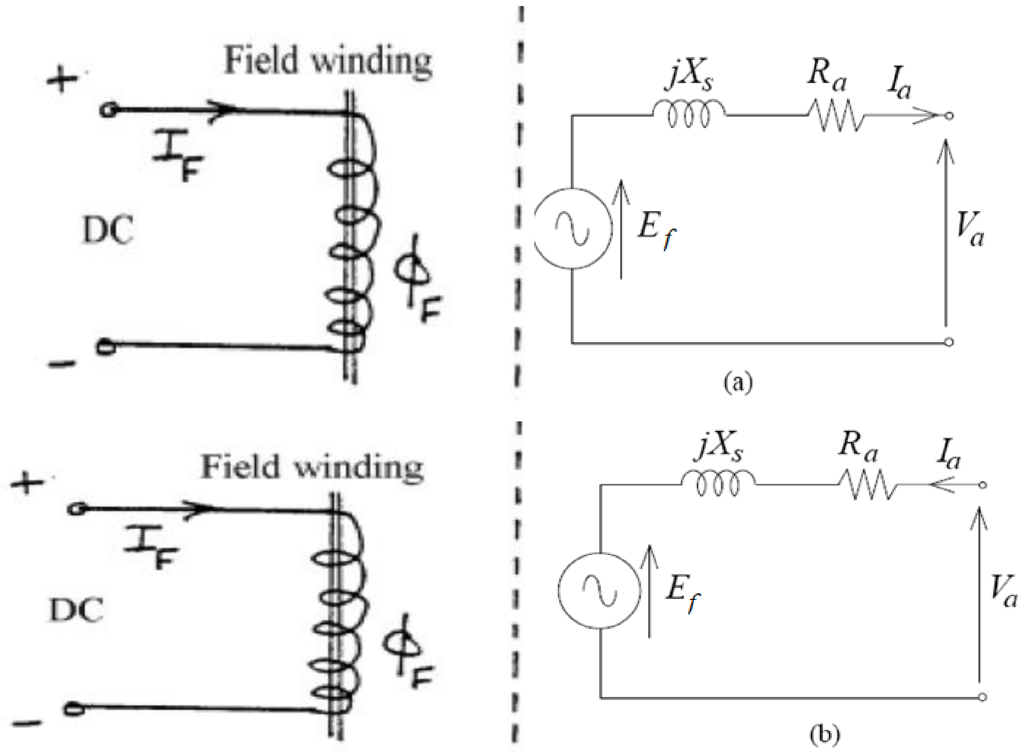


Fig. 3, equivalent circuit (a) synchronous Generator (b) synchronous Motor

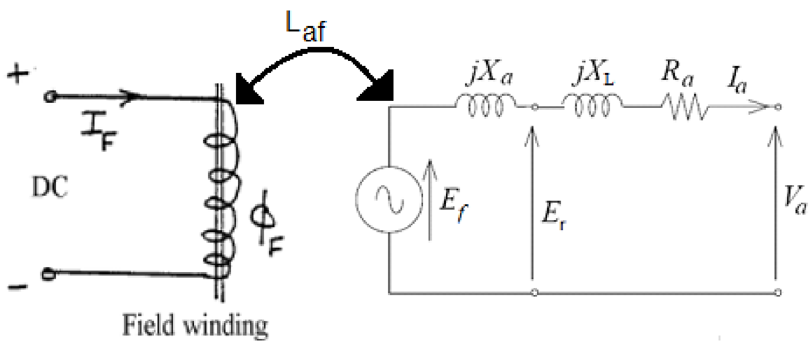


Fig. 4, equivalent circuit including leakage reactance and air-gap voltage

R_a armature resistance

X_L Leakage reactance (Leakage flux) saturation doesn't affect its value

X_a armature reactance (corresponding to Linkage flux) saturation affects its value

E_f the field voltage (Exciter DC voltage induced at the armature side)

E_r the air-gap voltage which is the resultant between E_f and E_a

The leakage reactance X_L and the armature reactance X_a may be combined to give synchronous reactance X_S.

$$X_S = X_L + X_a$$

Also, $R_a + JX_S$ is called synchronous impedance Z_S

If the mutual inductance between the rotor and armature (L_{af}) is considered as shown in Fig. 4, then the rms value of the induced voltage E_f can be given as:

$$E_f = \frac{2\pi f \; L_{af} \; I_f}{\sqrt{2}}$$

Therefore, the excitation current (I_f) can be calculated as:

$$I_f = \frac{\sqrt{2} \; E_f}{2\pi f \; L_{af}}$$

5. Phasor Diagram of Unsaturated Cylindrical Alternators

5.1 Lagging power factor

Assume that the synchronous generator is loaded with a lagging power factor load.

From the phasor diagram shown in Fig. 5, it is clear that the terminal voltage is decreased from its no-load value E_f to its loaded value V_a (for a lagging power factor). This is because of: Drop due to armature resistance, IR_a **&** drop due to leakage reactance, IX_L and drop due to armature reaction IX_a.

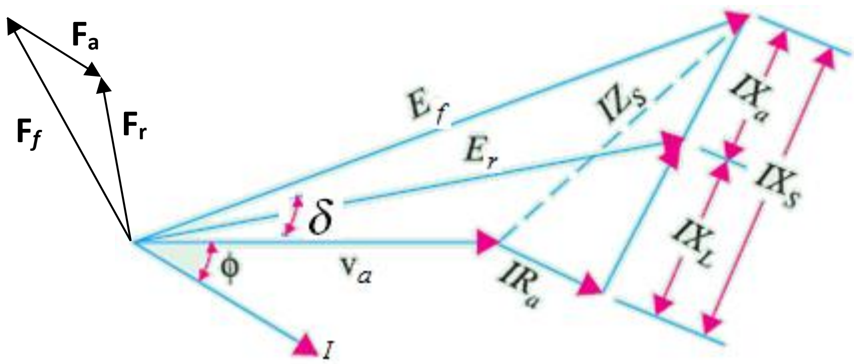


Fig. 5, Phasor diagram for synchronous generator (p.f. Lag)

The angle (δ) between the no-load voltage (E_f) and the terminal voltage (V_a) is called the load angle or (power angle) and it is positive value in case of alternators.

The DC voltage (Excitation voltage) produces a flux (Φ_f) or (field mmf F_f). If the armature circuit is closed by an electric load, the armature reaction (Φ_a) or (armature mmf F_a) is produced. These two fluxes may support each other or oppose each other depend on the load power factor to produce the air-gap or resultant flux (Φ_r) or (resultant mmf F_r).

From the phasor diagram shown in Fig. 5,

Since $F_r < F_f$ this means that F_a oppose F_f

Since $V_a < E_f$ this is called over-excited alternator

5.2 Leading power factor

Assume that the synchronous generator is loaded with a leading power factor load.

From the phasor diagram shown in Fig. 6, it is clear that the terminal voltage is increased from its no-load value E_f

$$E_f = V + I_a(R_a + JX_s)$$

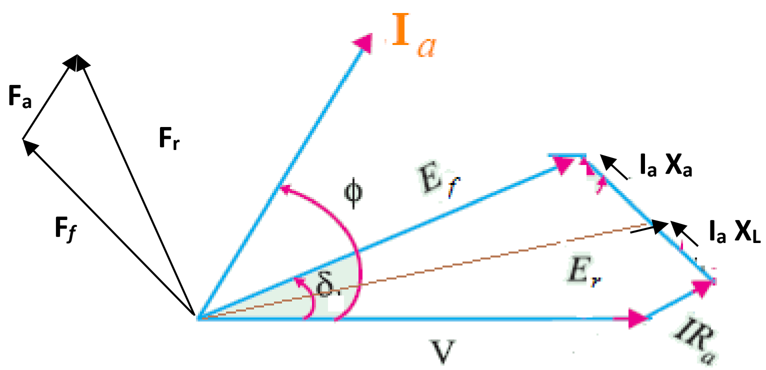


Fig. 6, Phasor diagram for synchronous generator (p.f. Lead)

From the phasor diagram shown in Fig. 6,

Since $F_r > F_f$ this means that F_a support F_f

Since $V_a > E_f$ this is called under-excited alternator

5.3 Unity power factor

Assume that the synchronous generator is loaded with a unity power factor load.

From the phasor diagram shown in Fig. 7, it is clear that the terminal voltage is decreased from its no-load value E_f (similar to lagging power factor)

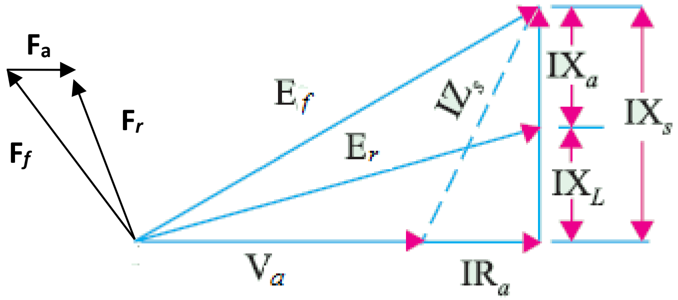


Fig. 7, Phasor diagram for synchronous generator (p.f. unity)

From the phasor diagram shown in Fig. 7,

Since $F_r < F_f$ this means that F_a oppose F_f

Since $V_a < E_f$ this is called over-excited alternator

6. Analytical Representation of Phasor Diagram

Consider the phasor diagram of 3-ph alternator at lagging p.f. as shown in Fig. 8-(a).

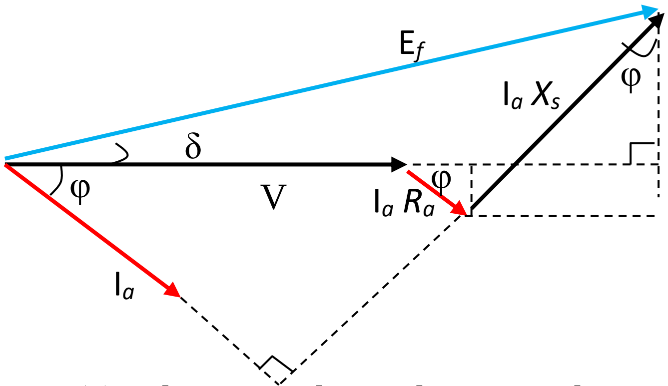


Fig. 8-(a), Alternator phasor diagram at lagging p.f

We can describe this phasor diagram by two equations:

Horizontal Analysis:

$$E_f \cos(\delta) = V + I_a R_a \cos(\varphi) + I_a X_s \sin(\varphi) \ldots (1)$$

Vertical Analysis:

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) - I_a R_a \sin(\varphi) \ldots (2)$$

By dividing (2) over (1), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi) - I_a R_a \sin(\varphi)}{V + I_a R_a \cos(\varphi) + I_a X_s \sin(\varphi)}$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

Now, if R_a is neglected, the phasor diagram is shown in Fig. 8(b).

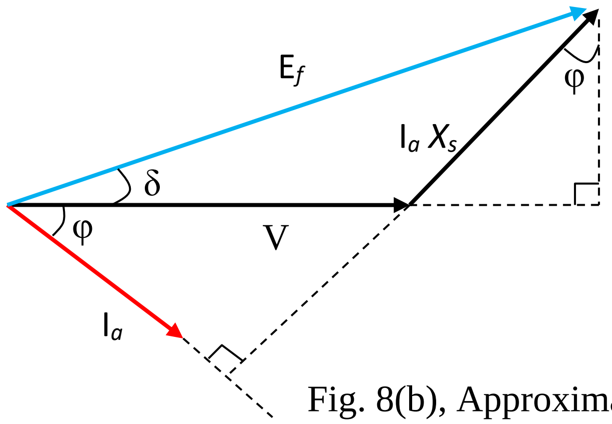


Fig. 8(b), Approximate phasor at lag p.f

Equations (1) and (2) can be rewritten by replacing R_a=0 as:

Horizontal Analysis:

$$E_f \cos(\delta) = V + I_a X_s \sin(\varphi) \ldots (3)$$

Vertical Analysis:

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) \ldots (4)$$

By dividing (4) over (3), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi)}{V + I_a X_s \sin(\varphi)} \ldots (5)$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

Example

A 4-pole, Y-connected, 3-phase synchronous generator is rated 250 MVA, its terminal voltage is 24 kV, the synchronous reactance is: 125%. • Calculate the synchronous reactance in ohm. • Calculate the rated current. • Calculate the induced voltage, E_f , at rated load and pf = 0.8 lag.

$$250 \times 10^6 = \sqrt{3} \times 24000 \times I_a \rightarrow I_a = 6014.0653A$$

$$Z_{base} = \frac{V_{phase}}{I_{phase}} = \frac{24000/\sqrt{3}}{6014.0653} = 2.304\ \Omega$$

Synchronous reactance in Ohm = X_s (in PU) × Z_{base} = 1.25×2.304=2.88Ω

The load angle can be obtained based on eqn. (5):

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi)}{V + I_a X_s \sin(\varphi)} = \frac{6014.0653 \times 2.88 \times 0.8}{13856.4065 + 6014.0653 \times 2.88 \times 0.6} = 0.57143$$

$$\delta = 29.745$$

By substituting with this value in eqn. (3) as:

$$E_f \times 0.868242 = 13856.4065 + 6014.0653 \times 2.88 \times 0.6$$

E_f = 27928.52

By the same way, we can describe the phasor diagram in case of leading power factor given in Fig. 9-(a) by two equations:

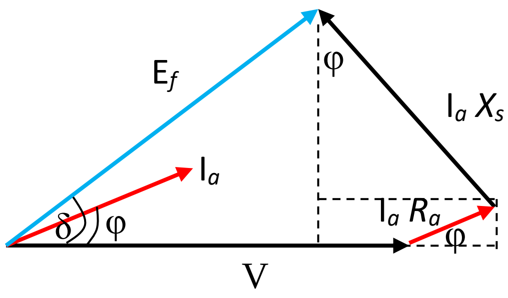


Fig. 9-(a), Alternator phasor diagram at leading p.f

<u>Horizontal Analysis:</u>

$$E_f \cos(\delta) = V + I_a R_a \cos(\varphi) - I_a X_s \sin(\varphi) \dots (6)$$

<u>Vertical Analysis:</u>

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) + I_a R_a \sin(\varphi) \dots (7)$$

By dividing (7) over (6), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi) + I_a R_a \sin(\varphi)}{V + I_a R_a \cos(\varphi) - I_a X_s \sin(\varphi)}$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

Now, if R_a is neglected, the phasor diagram is shown in Fig. 9(b).

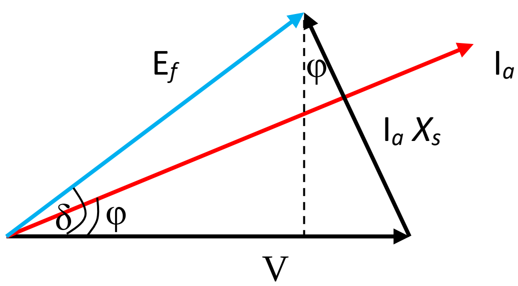


Fig. 9-b, Approximate phasor at leading p.f

Equations (6) and (7) can be rewritten by replacing R_a=0 as:

<u>Horizontal Analysis:</u>

$$E_f \cos(\delta) = V - I_a X_s \sin(\varphi) \dots (8)$$

Vertical Analysis:

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) \ldots (9)$$

By dividing (9) over (8), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi)}{V - I_a X_s \sin(\varphi)}$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

From the explained phasor diagrams given in Figs 5 to 9, we notice that V is always behind E_f, this means the power angle (δ) is always positive, and this is the remarkable notice on the phasor diagram of synchronous generators.

7. Experimental Determination of Circuit Parameters

In the per phase equivalent circuit model illustrated in section 1, there are three parameters need to be determined: armature winding resistance R_a, synchronous reactance X_s, and induced *emf* in the phase winding V_a. The phase winding resistance R_a can be determined by measuring DC resistance of the winding using volt-ampere method (DC test), while the synchronous reactance and the induced *emf* can be determined by the open circuit and short circuit tests.

7.1 DC Test

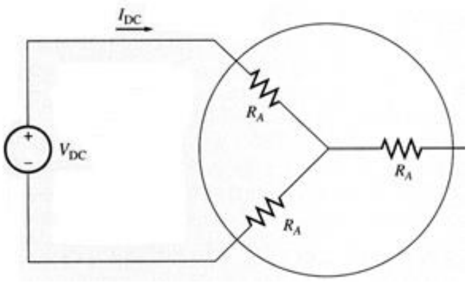
The purpose of the DC test is to determine R_a. A variable DC voltage source is connected between two stator terminals.

The DC source is adjusted to provide approximately rated stator current, and the resistance between the two stator leads is determined from the voltmeter and ammeter readings. Then

$$R_{DC} = \frac{V_{DC}}{I_{DC}}$$

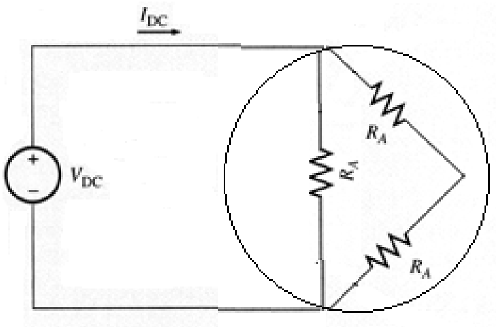
If the stator is Y-connected, the per phase stator resistance is

$$R_a = \frac{R_{DC}}{2} \times 1.15$$



If the stator is delta-connected, the per phase stator resistance is

$$R_a = \frac{3}{2} R_{DC} \times 1.15$$



Here, we take the skin effect of 15% to calculate the AC value of R_a

7.2 Open Circuit Test

Drive the synchronous machine at the synchronous speed using a prime mover when the stator windings are open circuited. Vary the rotor (field) winding current (I_f) , and measure stator winding terminal voltage (V). The relationship between the stator winding terminal voltage and the rotor field current obtained by the open circuit test is known as the ***open circuit characteristic (O.C.C.)*** of the synchronous machine as shown in Fig. 10.

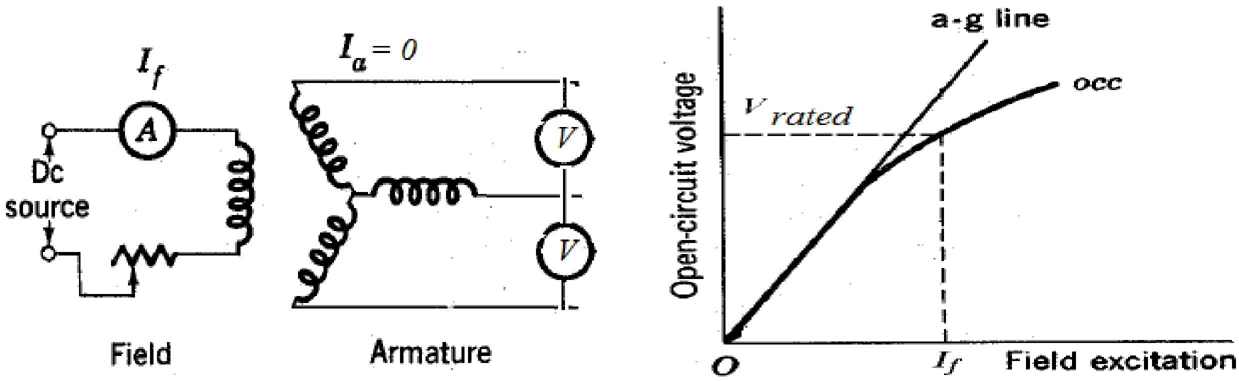


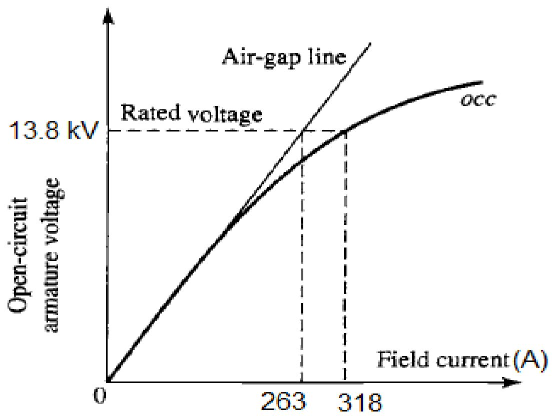
Fig. 10, Open-Circuit Characteristic (O.C.C.)

From the OCC shown in Fig. 10, the effects of magnetic saturation can be clearly seen; the characteristic bends downward with increasing the field current. As saturation of the magnetic material increases, the permeability decreases and as a result, the reluctance of the flux paths is increases and reduces the effectiveness of the field current in producing magnetic flux. As can be seen from Fig. 10, the open-circuit characteristic is initially linear as the field current is increased from zero. This portion of the curve (and its linear extension for higher values of field current) is known as the *air-gap line.* It represents the machine open-circuit voltage characteristic corresponding to unsaturated operation. Deviations of the actual open-circuit characteristic from this air-gap line are a measure of the degree of saturation in the machine.

Note that with the machine armature winding open-circuited, the terminal voltage is equal to the generated voltage E_f. Thus the open-circuit characteristic is a measurement of the relationship between the field current I_f and E_f. It can therefore provide a direct measurement of the field-to-armature mutual inductance L_{af}.

Example

An open-circuit test performed on a three-phase, 60-Hz synchronous generator shows that the rated open-circuit voltage of 13.8 kV is produced by a field current of 318 A. Extrapolation of the air-gap line from a complete set of measurements on the machine shows that the field current corresponding to 13.8 kV on the air-gap line is 263 A. Calculate the saturated and unsaturated values of L_{af}.



$E_f = 13800/\sqrt{3} = 7967.434$ V

To calculate the unsaturated value of L_{af}, we use the air-gap quantity of field current

$$L_{af}\Big|_{Unsaturated} = \frac{\sqrt{2}\,E_f}{2\pi f\ I_f\big|_{air\ gap}}$$

$$L_{af}\Big|_{Unsaturated} = \frac{\sqrt{2}\times 7967.434}{2\pi\times 60\times 263} = 113.644\ mH$$

To calculate the saturated value of L_{af}, we use the O.C quantity of field current

$$L_{af}\Big|_{Saturated} = \frac{\sqrt{2}\,E_f}{2\pi f\ I_f\big|_{open\ circuit}}$$

$$L_{af}\Big|_{Saturated} = \frac{\sqrt{2}\times 7967.434}{2\pi\times 60\times 318} = 93.989\ mH$$

The saturation reduces the magnetic coupling between field and armature windings by

$$\frac{113.644-93.989}{113.644}\times 100\% = 17.295\ \%$$

7.3 Short Circuit Test

Reduce the field current to a minimum value, using the field rheostat, and then open the field supply circuit breaker. Short the stator terminals of the machine together through three ammeters; Close the field circuit breaker; and raise the field current gradually to the value noted in the open circuit test at which the open circuit terminal voltage equals the rated voltage, while maintain the synchronous speed. Record the three stator currents. (This test should be carried out quickly since the stator currents may be greater than the rated value). Plot the relation between the field current and the armature current as shown in Fig. 11. Such kind of relation is called Short Circuit Characteristics (S.C.C.). It is clear from the S.C.C. shown in Fig. 11, the relation between the field current and the short-circuit armature current is straight line passing through origin. $E_f = I_a\,(R_a + jX_s)$

if R_a is negligible, I_a will lag E_f by nearly 90 elec. deg.

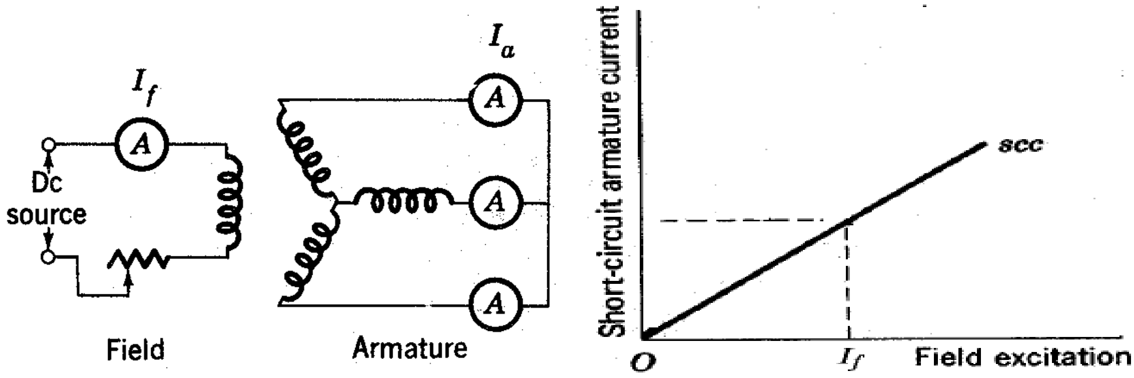


Fig. 11, Short-circuit characteristics S.C.C.

7.4 Calculation of synchronous reactance X_S

Following procedural steps are involved in this calculation:

1. O.C.C is plotted from the given data as shown in Fig. 10.

2. Similarly, S.C.C. is drawn from the data given by the short-circuit test as shown in Fig. 11. It is a straight line passing through the origin.

Both these curves are drawn on a common field-current base as shown in Fig. 12, from which, the value of Z_S is not constant but varies with saturation. At low saturation, its value is larger because the effect of a given armature ampere-turns is much more than at high saturation. Now, under short-circuit conditions, saturation is very low, because armature m.m.f. is directly demagnetising. Different values of Z_S corresponding to different values of field current are also plotted

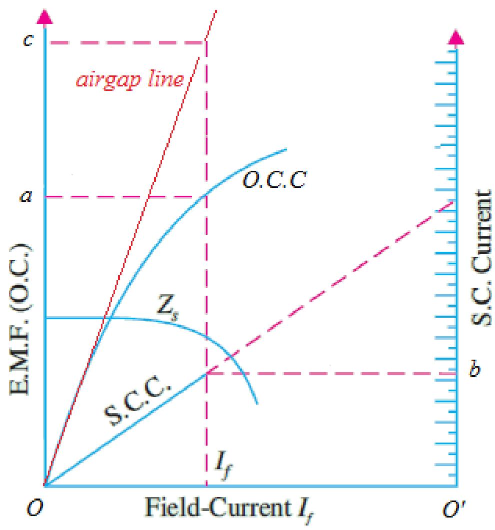


Fig. 12, Determination of saturated and unsaturated values of X_s using S.C.C. and O.C.C.

Consider a field current I_f. The O.C. voltage corresponding to this field current is *Oa*. When winding is short-circuited, the terminal voltage becomes zero. Hence, it may be assumed that the whole of this voltage *Oa* is being used to circulate the armature short-circuit current *O'b* against the synchronous impedance Z_S.

Based on both O.C.C. and S.C.C. given in Fig. 12, at any convenient field current (I_f), the saturated synchronous impedance Z_S can be calculated by:

$$Z_S = \left.\frac{O.C.Voltage}{S.C.Current}\right]_{at\ same\ field\ current}$$

$$Z_S = \frac{Oa\ (from\ O.C.C.)}{O'b\ (from\ S.C.C.)}$$

$$X_S = \sqrt{{Z_S}^2 - {R_a}^2}\ (Saturated\ Value)$$

The unsaturated synchronous impedance Z_S can be calculated by:

$$Z_S = \left.\frac{O.C.Voltage}{S.C.Current}\right]_{at\ same\ field\ current}$$

$$Z_S = \frac{Oc\ (from\ airgap\ line)}{O'b\ (from\ S.C.C.)}$$

$$X_S = \sqrt{{Z_S}^2 - {R_a}^2}\ (unsaturated\ Value)$$

The resistive element of the machine can simply be found from the DC test explained before. Value obtained in this test (R_a) may increase the X_S accuracy.

* **<u>Notes on this method</u>**

The important comment on the determination of saturated and unsaturated values of X_s using S.C.C. and O.C.C. is as follows;

E_f is taken from the OCC whereby the core would be *partially saturated* for large field currents, while I_a is taken from the SCC where the core is *not saturated* at all field currents. Therefore E_f value taken during the OCC may not be the same E_f value in the SCC test. Hence the value of X_S is only an approximate.

Hence to gain better accuracy, the test should be done at low field currents which looks at the linear region of the OCC test.

Example 1

From the following tests, determine the synchronous reactance, assuming R_a *is 0.8Ω.*

S.C.C: a current of 100 A is produced on short-circuit by a field excitation of 2.5A.

O.C.C: An e.m.f. of 500 V (phase) is produced on open-circuit by the same excitation.

$$Z_S = \frac{500}{100} = 5\ \Omega$$

$$X_S = \sqrt{{Z_S}^2 - {R_a}^2} = \sqrt{5^2 - 0.8^2} = 4.936\ \Omega \quad \text{(Saturated value)}$$

7.5 Short Circuit Ratio (SCR)

SCR is defined as the ratio of the field current required for the rated voltage at open circuit to the field current required for rated armature current at short circuit. Based on Fig. 13, the SCR can be obtained as:

$$SCR = \frac{Of'}{Of''}$$

The saturated synchronous reactance (in per unit) can be calculated as the inverse of SCR

$$X_S(saturated\ in\ PU) = \frac{1}{SCR} = \frac{Of''}{Of'}$$

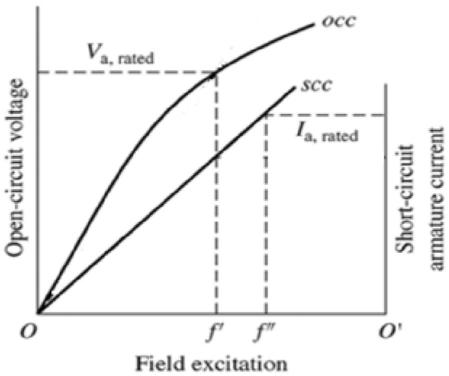


Fig. 13, Short Circuit Ratio (S.C.R.)

Example 2

The following data are taken from the open- and short-circuit characteristics of a 45 kVA, 3-ph, Y-connected, 220 (line-to-line), 6-poles, 60 Hz synchronous machine:

OCC: line-to-line voltage = 220 V & Field current = 2.84 A

SCC: Armature Current = 118 A & Field current = 2.2 A

Air-gap line: line-to-line voltage = 202 V & Field current = 2.2 A

Calculate the SCR and Compute the saturated and unsaturated value of synchronous reactance at rated voltage in Ohms and in per unit.

V_a (rated) =220 Line-to-line → from OCC, the corresponding field current = 2.84 A

I_a (rated) = $\frac{45000}{\sqrt{3}\,220} = 118\ A$ → from SCC, the corresponding field current = 2.2 A

SCR = $\frac{2.84}{2.2} = 1.29$

X_S (Saturated) = 1/SCR = 1/1.29 = 0.775 P.U.

$$X_S(Base) = \frac{V_{rated}}{I_{rated}} = \frac{220/\sqrt{3}}{118} = 1.0764\ \text{Ohm}$$

X_S (Saturated) = 0.775 * 1.0764 = 0.8342 Ohm

From SCC, at Field Current of 2.2 A gives armature current of 118 A

From Air-gap Line, at Field Current of 2.2 A gives armature voltage of 116.6 V/phase

X_S (unsaturated) = 116.6/118 = 0.988 Ohm

X_S (unsaturated) = 0.988/1.0764 = 0.918 P.U.

Example (3)

A 200 kVA, 480-V, 60-Hz, 4-pole, Y-Connected synchronous generator with a rated field current of 5 A was tested and the following data was taken.

a) from OC test – terminal voltage = 540 V at rated field current
b) from SC test – line current = 300A at rated field current
c) from DC test – DC voltage of 10 V applied to two terminals, a current of 25 A was measured.

1. Calculate the speed of rotation in rpm
2. Calculate the saturated equivalent circuit parameters (armature resistance and synchronous reactance) in Ohms and PU.

V_{rated} = 480/√3 = 277.13 V and $I_{a\ rated}$ = 200000/(√3 * 480) = 240.56 A

Z_{base} = V_{rated} / $I_{a\ rated}$ = 277.13 / 240.56 = 1.152 Ω

The synchronous speed n_s = 60 *f* / P = 60*60/2 = 3600/2 = 1800 rpm

<u>From DC Test</u>

R_{DC} = 10/25 = 0.4 Ω

R_a = (0.4/2)*1.15 = 0.23 Ω

R_a = 0.23 / 1.152 = 0.2 PU.

From OC Test

Rated field current produce rated voltage of 540/√3 = 311.77 V

From SC Test

Rated field current produce rated current = 300 A

Saturated Z_S = 311.77/300 = 1.04 Ω

Saturated Z_S = 1.04 / 1.152 = 0.903 PU.

$$X_s = \sqrt{Z_S{}^2 - R_a{}^2} = \sqrt{1.04^2 - 0.23^2} = 1.0287\ \Omega$$

$$X_s = \frac{1.0287}{1.152} = 0.893\ \text{PU.}$$

Example (4)

The following readings are taken from the results of open- and short-circuit test on a 5000-kVA, 4160-V, 3-phase, 4-pole, 1800-rpm synchronous motor driven at rated speed:

Field current, A	169	192
Armature current, short-circuit test, A	694	790
Line voltage, open-circuit test, V	3920	4160
Line voltage, air-gap line, open-circuit test, V	4640	5270

The armature resistance is 11 mΩ/phase. The armature leakage reactance is 0.12 PU. Find (a) the short-circuit ratio,

(b) Saturated value of the armature reactance in ohms per phase and per unit,

(c) Unsaturated armature reactance in per unit and in ohms per phase.

V_a (rated) =4160 Line-to-line → from OCC, the corresponding field current = 192 A

I_a (rated) = $\frac{5000000}{\sqrt{3}\,4160} = 694\ A$ → from SCC, the corresponding field current = 169 A

$$SCR = \frac{192}{169} = 1.1361$$

$$Z_{base} = \frac{4160/\sqrt{3}}{694} = 3.460774\ \Omega$$

$X_L = 0.12 \times Z_{base} = 0.4153\Omega$

b) Saturated synchronous impedance Z_S can be calculated by:

$$Z_S = \frac{O.C.Voltage}{S.C.Current}\Big]_{at\ same\ field\ current}$$

$$Z_S = \frac{3920/\sqrt{3}}{694} = 3.261114\ \Omega$$

$$Z_S = \frac{4160/\sqrt{3}}{790} = 3.040224\ \Omega$$

The average value of the saturated Z_s can be calculated as:

$$Z_s = \frac{3.261114 + 3.040224}{2} = 3.15067\ \Omega$$

$$X_S(Saturated\ Value) = \sqrt{{Z_S}^2 - {R_a}^2} = \sqrt{3.15067^2 - 0.011^2} = 3.15065\ \Omega$$

$$X_a(Saturated\ Value) = X_S - X_L = 3.15065 - 0.4153 = 2.73535\ \Omega$$

$$X_a(Saturated\ Value) = \frac{X_a}{Z_{base}} = \frac{2.73535}{3.460774} = 0.7904\ PU$$

c) Unsaturated synchronous impedance Z_S can be calculated by:

$$Z_S = \frac{air - gap\ Voltage}{S.C.Current}\Big]_{at\ same\ field\ current}$$

$$Z_S = \frac{4640/\sqrt{3}}{694} = 3.8601\ \Omega$$

$$Z_S = \frac{5270/\sqrt{3}}{790} = 3.85144\ \Omega$$

The average value of the saturated Z_s can be calculated as:

$$Z_s = \frac{3.8601 + 3.85144}{2} = 3.85577\ \Omega$$

$$X_S(Saturated\ Value) = \sqrt{{Z_S}^2 - {R_a}^2} = \sqrt{3.85577^2 - 0.011^2} = 3.85575\ \Omega$$

$$X_a(Saturated\ Value) = X_S - X_L = 3.85575 - 0.4153 = 3.44045\ \Omega$$

$$X_a(Saturated\ Value) = \frac{X_a}{Z_{base}} = \frac{3.44045}{3.460774} = 0.994\ PU$$

7.6 Obtaining the phasor diagram from OCC and SCC at saturation condition

In case of saturation, X_s is variable (unknown). But all information to calculate E_r is given, therefore,

$$E_r = V + I_a(R_a + j\, X_L)$$

From OCC at E_r we get the corresponding F_r which is $\perp$ E_r and lead by 90

from SCC at I_a find the corresponding F_a where F_a is drawn in the same direction of I_a

Since $F_r = F_f + F_a$, we can obtain F_f

Again from OCC at the value of F_f, we can obtain E_f, The direction of E_f is perpendicular to F_f and lag by 90. These steps are given in Fig. 14.

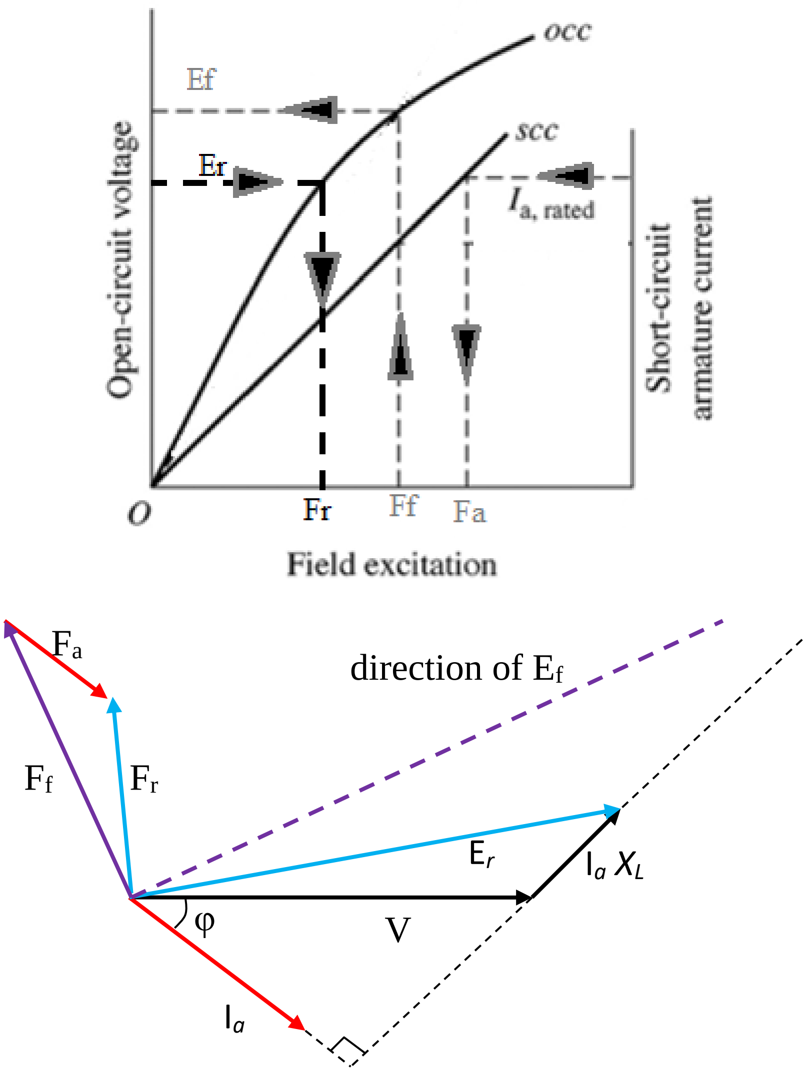


Fig. 14, Phasor diagram from OCC and SCC

7.7 Voltage regulation

It is clear that with change in load, there is a change in terminal voltage of an alternator. The magnitude of this change depends ***not only on the load but also on the load power factor.*** The voltage regulation of an alternator is defined as "the rise in voltage when full-load is removed (field excitation and speed remaining the same) divided by the rated terminal voltage."

In case of lagging and unity power factors, the regulation is positive (up)

$$\% \, regulation \, (up) = \frac{E_f - V_a}{V_a} * 100$$

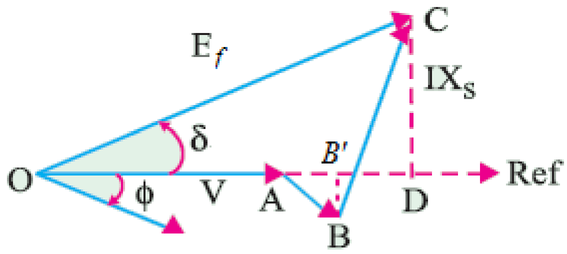
In case of leading power factor, the regulation is negative (down).

$$\% \, regulation \, (down) = \frac{E_f - V_a}{V_a} * 100$$

Example (5)

A 1000 kVA, 3300-V, 3-phase, star-connected alternator delivers full-load current at rated voltage at 0.80 p. f. Lagging. The resistance and synchronous reactance of the machine per phase are 0.5 ohm and 5 ohms respectively. Estimate the terminal voltage for the same excitation and same load current at 0.80 p. f. leading.

In case of lagging power factor



Rated phase voltage (V) = 3300/√3 = 1905.26 volt

Rated phase current (I_a) = $\frac{1000\times10^3}{\sqrt{3}\times3300} = 174.95$ Amp.

I_aR_a = 174.95*0.5 = 87.475 volt

I_aX_S = 174.95*5.0 = 874.75 volt

Component of E_f along the horizontal axis OD = OA + AB' + B'D

OD = V + AB cos(ϕ) + BC sin(ϕ) = 1905.26 + 87.475*0.8 + 874.75*0.6 = 2500.09

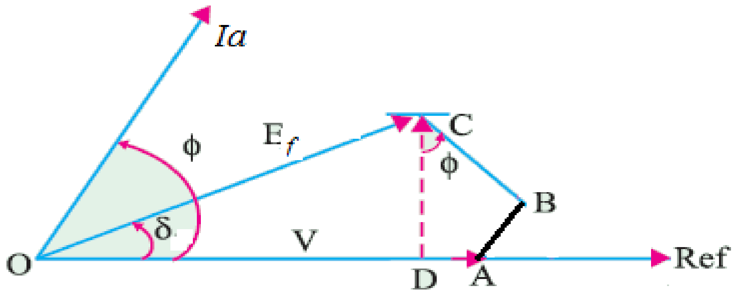
Component of E_f along the vertical axis DC = BC cos(ϕ) – AB sin(ϕ)

DC = 874.75*0.8 – 87.475*0.6 = 647.315

$E_f = \sqrt{2500.09^2 + 647.315^2} = 2582.53$ volt

Load angle $\delta = tan^{-1}\frac{DC}{OD} = tan^{-1}\frac{647.315}{2500.09} = 14.52$

In case of leading power factor



Since the excitation is kept constant, this means E_f is the same in magnitude, and the alternator delivering rated current at 0.80 leading p.f., the phasor diagram is to be drawn to evaluate *V*.

Construction of the phasor diagram starts with marking the reference. Take a point *A* which is the terminating point of phasor *V* which starts from *O*. Point *O* is the point yet to be marked, for which the other phasors have to be drawn. Draw AB parallel to the current I_a. Then draw *BC* perpendicular to *AB*. From *C*, draw an arc of length *E*, *i.e.* 2582.53 volts to intersect with the reference line (ref) to locate *O*. Therefore the terminal voltage OA represents the terminal voltage (V).

AB = I_aR_a = 87.475 volt with angle 36.8 and BC = I_aX_S = 874.75 volt $\perp I_a$

From the phasor diagram, V = 2925 Volt

Example (6)

The following test results were obtained on a 275-kW, 3-ph, 6600-V non-salient pole type generator. Open-circuit characteristic :

Phase Voltage (V)	*3233*	*3810*	*4180*	*4677*
Field Current (A)	*46.5*	*58*	*67.5*	*96*

Short-circuit characteristic : Stator current 35 A with an exciting current of 50 A. Leakage reactance drop on full-load = 8%.

Neglect armature resistance. Calculate as accurately as possible the exciting current (for full-load) at power factor 0.8 lagging.

Rated phase voltage $V_a = 6600/\sqrt{3} = 3810.51178$ V.

Rated Current $I_a = 275\times10^3 / (3\times3810.5\times0.8) = 30.07033$ A

$I_aX_L = 8\% = 0.08$ P.U. $= 0.08 \times 3810.51178 = 304.8409$ V

$X_L = 304.8409/30.07033 = 10.1376\ \Omega$

From the phasor diagram shown in Fig. 16, draw the rated voltage as a reference OA.

Then draw $I_aX_L = 304.8409$ V $\perp I_a$ and represented by the length AB.

<u>E_r can be obtained graphically by</u> the length OB = 4010 V.

<u>E_r can be calculated analytically by:</u>

$$E_r = \frac{6600}{\sqrt{3}} + 304.8409\angle(90 - 36.8699) = 4000.856\angle 3.4946\ V$$

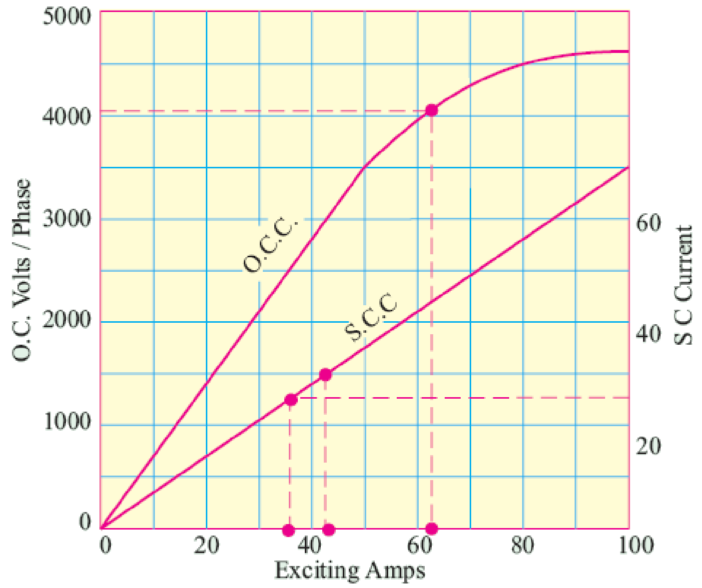


Fig. 15, OCC and SCC of example 5

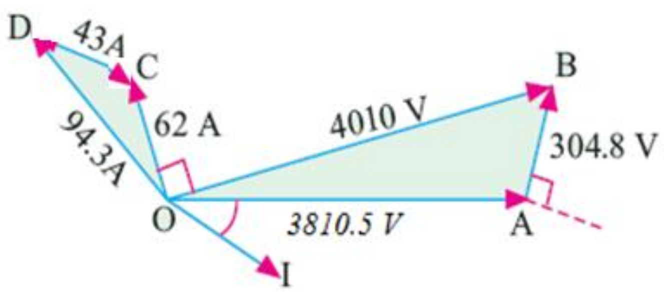


Fig. 16, phasor diagram of example 5

From the OCC shown in Fig. 16, at voltage of 4010, we can get F_r = 62 A and represented by OC ⊥ OB.

From the SCC, shown in Fig. 15, at full load current of 30 A, the corresponding field current F_a = 43 A that is represented by the line DC // I_a and in the same direction.

➔ (hint) No need to draw the SCC as it is represented by a straight line.

$$I_a = 35\text{ A} \qquad \rightarrow I_f = 50\text{ A}$$

$$I_a = 30.07033\text{ A} \rightarrow F_a = ??\text{ A}$$

$$F_a = 50 \times (30.07033/35) = 42.9576\text{ A}$$

From the MMF triangle ODC, the value of F_f that is represented by OD = 94.3 A

<u>F_f can be obtained analytically by:</u>

$$F_f = F_r - F_a = 62\angle(90 + 3.4946) - 42.9576\angle - 36.8699 = 95.599\angle 113.5164$$

7.8. Zero Power Factor Test or Potier Triangle Method

This method is based on the separation of armature-leakage reactance X_L and the armature reaction effects X_a. Hence, it gives more accurate results. The experimental data required is

(i) No-load curve Or O.C.C. and

(ii) Full-load, zero-power factor curve (Z.P.F.C), also called wattless load characteristic. It is the curve of terminal volts against excitation when armature is delivering F.L. current at zero p.f. (when loaded by a pure inductive load for example). Both OCC & ZPFC are shown in Fig. 17.

Point *A*, at which (V=0) and the current flow is the full load current, on Z.P.F.C is obtained from a short-circuit test with full-load armature current. Hence, *OA* represents field current which is equal and opposite to the demagnetising armature reaction and for balancing leakage reactance drop at full-load.

From *B*, draw *BH* equal to and parallel to *OA*. From *H*, *HD* is drawn parallel to the air gap line OC. Hence, we get point *D* on the O.C.C. curve. Connect the two points DB to get the triangle *BHD* which is known as ***Potier triangle***. This triangle is constant for a given armature current and hence can be transferred to give us other points like *M*, *L* etc. Draw *DE* perpendicular to *BH*. The length *DE* represents the voltage drop due to armature leakage reactance X_L i.e. I_aX_L. *BE* gives field current necessary to overcome demagnetising effect of armature reaction at full load and *EH* for balancing the armature leakage reactance drop *DE*.

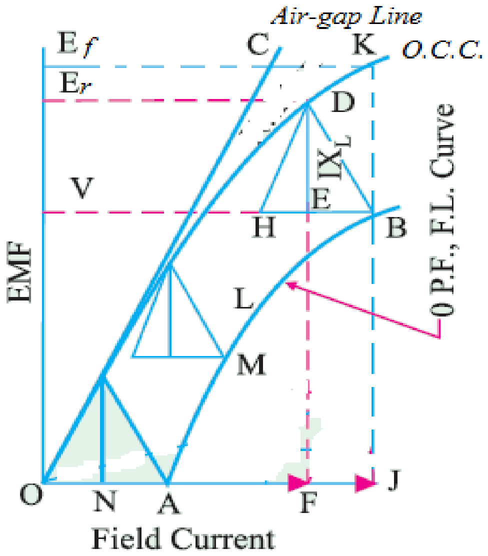


Fig. 17, Potier Triangle

Let *V* be the terminal voltage on full-load, then if we add to it vectorially the voltage drop due to armature leakage reactance alone (neglecting R_a), then we get voltage E_r = *DF*. Obviously, field excitation corresponding to E_r is given by *OF*. *NA* (= *BE*) represents the field current needed to overcome armature reaction. Hence, if we add *NA* vectorially to *OF*, we get excitation(OJ) required to produce E_f whose value from *O.C.C* curve (JK).

Example (7)

A 3-phase, 6000-V alternator has the following O.C.C. at normal speed:

Field Current (A)	14	18	23	30	43
Terminal Voltage (V)	4000	5000	6000	7000	8000

With armature short-circuited and full-load current flowing, the field current is 17 A. Also when the machine is supplying full-load of 2,000 kVA at zero power factor, the field current is 42.5 A and the terminal voltage is 6000 V.

Determine the field current required when the machine is supplying the full-load at 0.8 p.f. lagging.

The O.C.C. at phase voltage is

Field Current (A)	14	18	23	30	43
Phase Voltage (V)	2309.4	2886.75	3464.1	4041.45	4618.8

We have two points on Z.P.F.C,

Field Current (A)	17	42.5
Phase Voltage (V)	0	3464.1

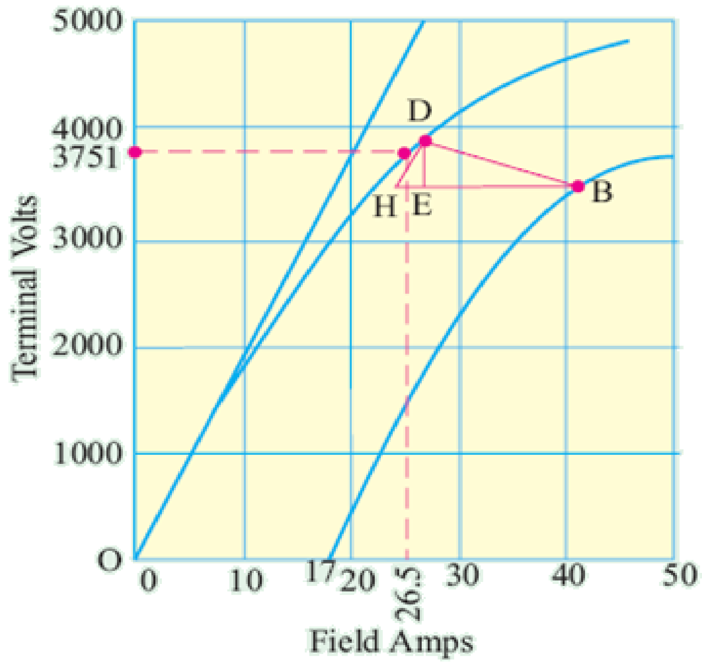


Fig. 18 OCC and ZPFC for example 6

From both the OCC and ZPFC we can obtain Potier triangle BHD as shown in Fig. 18.

From Potier triangle, DE represents I_aX_L = 450 V.

From the phasor diagram shown in Fig. 19, we can obtain E_r graphically (OB) or mathematically as:

$$E_r = \sqrt{(V_a + I_aX_L \sin(\varphi))^2 + (I_aX_L \cos(\varphi))^2}$$

$$E_r = \sqrt{(3464.1 + 450 \times 0.6)^2 + (450 \times 0.8)^2} = 3751.41\ V$$

From the OCC at 3751 V, we get the corresponding field current (F_r) of 26.5 A.

This field current (OC = 26.5) is drawn ⊥ OB

From Potier triangle we can get the field current (F_a corresponding to I_aX_a) which is BE = 14.5 A. This value is drawn in Fig. 19 as the length DC parallel to the armature current and in the same direction.

Therefore, from the MMF triangle ODC, the field current (F_f) OD = 37.2 A.

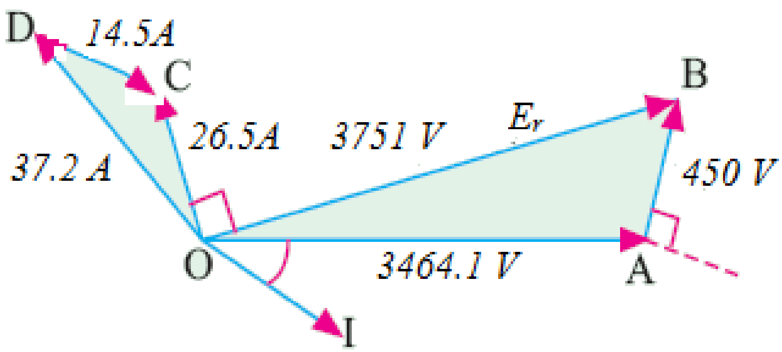


Fig. 19, Phasor diagram of example 6

Example (8)

An 11-kV, 1000-kVA, 3-phase, Y-connected alternator has a resistance of 2 Ω per phase. The open-circuit and full-load zero power factor characteristics are given below.

Find the voltage regulation of the alternator for full load current at 0.8 p.f. lagging by Potier method.

Field Current (A)	40	50	110	140	180
OCC line voltage	5800	7000	12500	13750	15000
ZPF line voltage	0	1500	8500	10500	12500

The phase voltages used in OCC and ZPFC are as in the following table

Field Current (A)	40	50	110	140	180
OCC phase voltage	3348.63	4041.45	7216.88	7938.57	8660.25
ZPF phase voltage	0	866	4907.48	6062.18	7216.88

The full-load current $I_a = 1000{*}10^3 /(\sqrt{3} \times 11{\times}10^3) = 52.49$ A

Phase voltage $V_a = 11{\times}10^3 / \sqrt{3} = 6350.85$ V

Potier triangle ACB is drawn as shown in Fig. 20. Where AC = 40 A. CB is drawn parallel to the air gap line. BD ⊥ AC.

BD, the leakage reactance drop I_aX_L = BD = 1000 V (by measurement)

AD = 30 A (armature reaction F_a)

We draw the phasor diagram of the alternator at 0.8 pf lag as shown in Fig. 18.

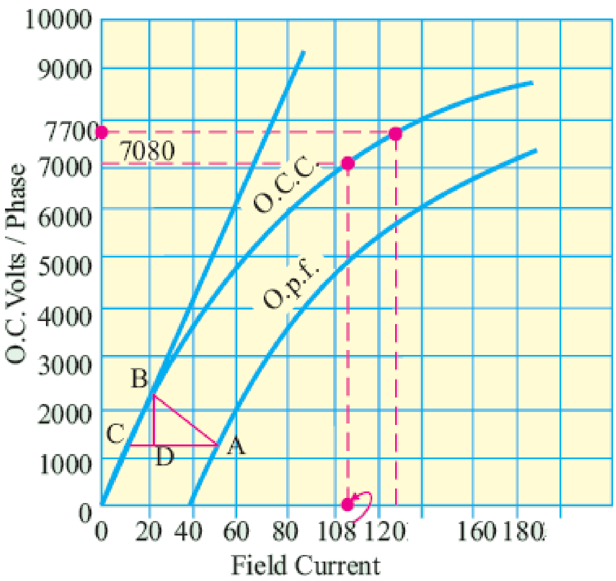


Fig. 20, Potier triangle of example 7

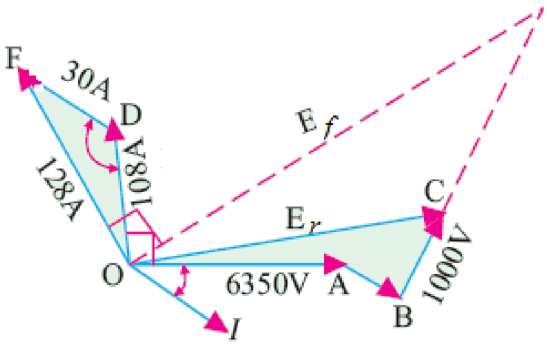


Fig. 21, Phasor diagram of example 7

Draw I_aR_a = 52.49*2 = 105 V represented by the length AB // I_a

Then draw I_aX_L = 1000 V represented by the length BC ⊥ I_a

Graphically, we can obtain the air gap voltage E_r = 7080 V

From OCC at 7080 V, the corresponding field current (F_r) is 108 A that is drawn ⊥ E_r as the line OD in Fig. 21.

From potier triangle F_a = 30 A and represented by the line FD // I_a and in the same direction.

From the MMF triangle OFD, the length OF that represent F_f is obtained = 128 A.

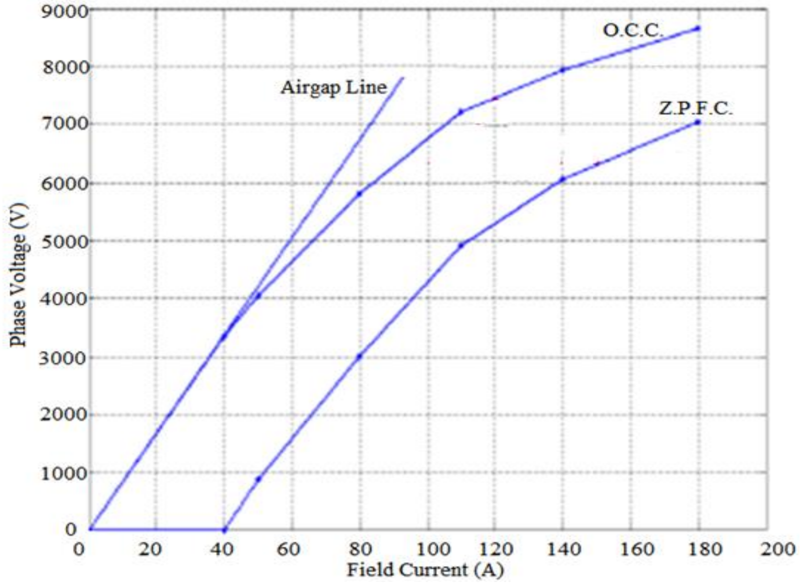
From the OCC at field current = 128, the corresponding voltage (E_f) = 7700 V.

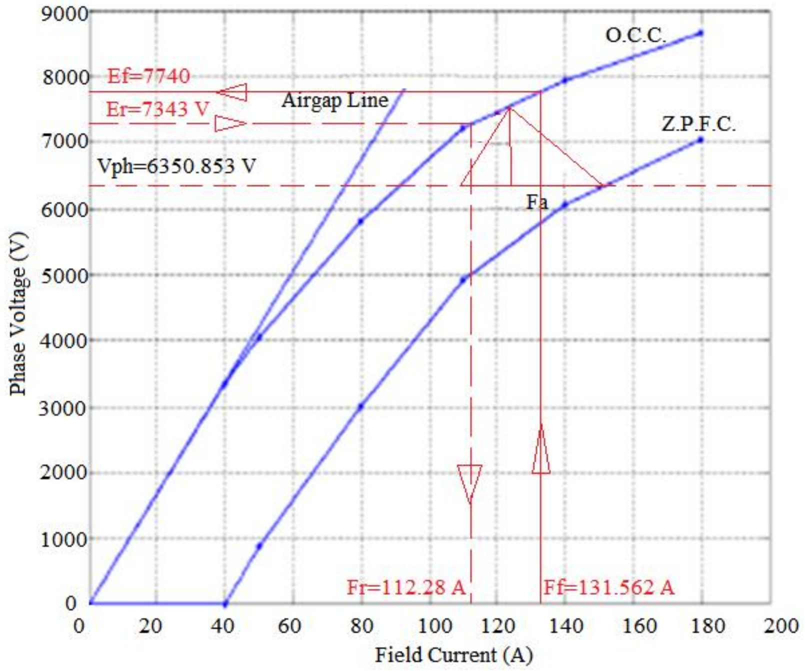
$$Regulation(up) = \frac{E_f - V_a}{V_a} = \frac{7700 - 6350}{6350} = 0.2126 = 21.26\%$$

Note: in example 5, the field excitation of 43 A is assumed to be used for balancing armature reaction F_a. In fact, a part of this value is used for balancing armature leakage drop of 304.8 V as explained in the Potier triangle method. Therefore, this is considered as inaccurate solution.

Example (9)

A 3000-kVA, 11-kV, 3-phase, star-connected alternator has resistance of 2 Ω/phase. The open circuit and the zero power factor characteristics are shown below. Based on Potier triangle, calculate: a) The leakage reactance in Ohm per phase.
b) The full-load voltage regulation at 0.8 p.f. lag.





a) From Potier triangle: $I_a X_L$ = 1.3cm = 1170 V, also F_a = 1.5 cm = 26.32 A

$I_a = \frac{3000\times10^3}{\sqrt{3}\times11000} = 157.4562\ A$, Φ = 36.87

The leakage reactance X_L = 1170/157.4562 = 7.431 Ω

b) $E_r = \frac{11000}{\sqrt{3}} + 157.4562\angle-36.87(2) + 1170\angle(90-36.87) =$

$7342.9\angle5.839$

From OCC at E_r = 7343, we obtain F_r = 6.4 cm = 112.28 A and lead E_r by 90

$$F_r = 112.28\angle95.839$$

As we know, $F_r = F_f + F_a$

$F_f = F_r – F_a$ = 112.28∠(5.839+90) – 26.32∠-36.87 = 131.562∠104.2924 A

From OCC at F_f = 131.562 A = 7.5 cm, we can obtain E_f = 8.6 cm = 7740 V. and lag behind F_f by 90

E_f = 7740 ∠(104.2924-90) = 7740∠14.2924

$$\epsilon = \frac{E_f - V}{V} = \frac{7740 - 6350.853}{6350.853} \times 100 = 21.8734\%\ (up)$$

8. Synchronous Motor

A synchronous motor, shown in Fig. 22, is electrically identical with an alternator or AC generator. In fact, a given synchronous machine may be used, at least theoretically, as an alternator, when driven mechanically or as a motor, when driven electrically, just as in the case of DC machines. Most synchronous motors are rated between 150 kW and 15 MW and run at speeds ranging from 150 to 1800 r.p.m.

The following characteristic features of a synchronous motor:

1. It runs either at synchronous speed or not at all *i.e.* while running it maintains a constant speed. The only way to change its speed is to vary the supply frequency.

2. It is not inherently self-starting. It has to be run up to synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.

3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

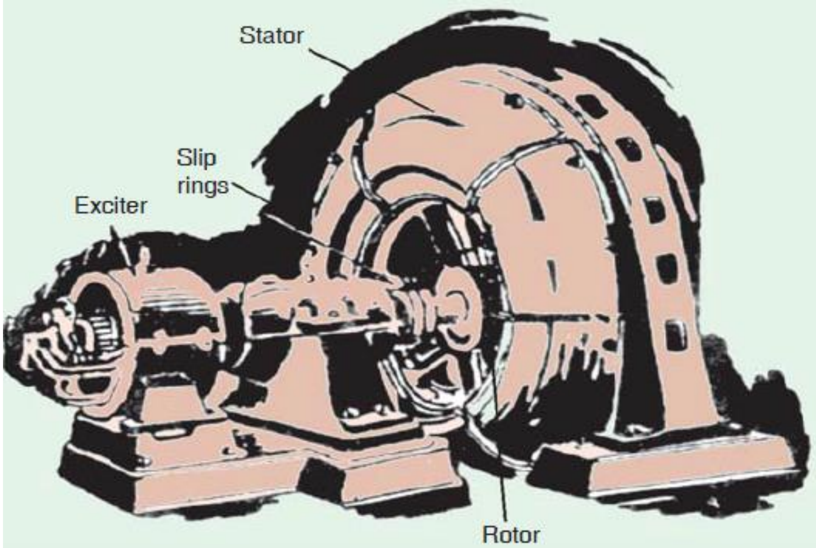


Fig. 22, Synchronous motor

8.1 Principle of Operation

when a 3-φ armature winding is fed by a 3-φ supply, then a magnetic flux of constant magnitude but rotating at synchronous speed, is produced. Consider a synchronous motor shown in Fig. 23, in which the stator has two poles (for simplicity) marked N_S and S_S and rotating at synchronous speed in clockwise direction. At certain instant, assume the position of the stator poles are situated at points *A* and *B* With the rotor poles marked N and S are located at the position shown in Fig. 24-a. The rotor and stator poles will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.

Half a period later, stator poles, having rotated around, interchange their positions *i.e.* N_S is at point *B* and S_S at point *A* as shown in Fig. 24-b. Under these conditions, N_S attracts *S* and S_S attracts *N*. Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction). Hence, we find that due to continuous and rapid rotation of stator poles, the rotor is subjected to a torque which is rapidly reversing *i.e.*, in quick succession, the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction. Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, with the result that it remains stationary.

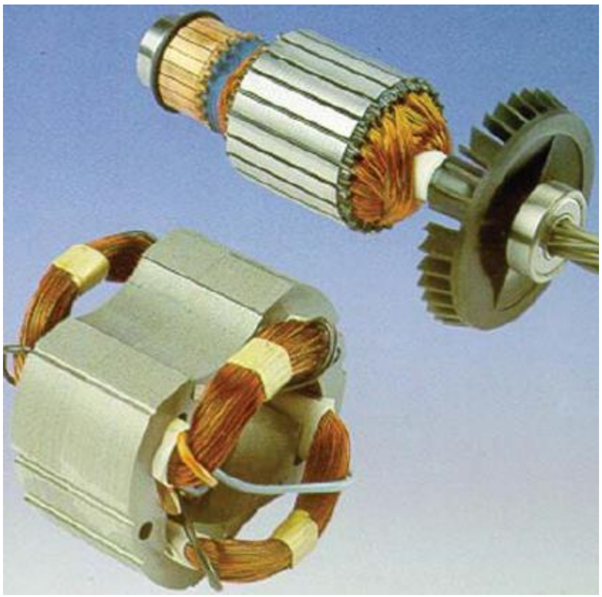
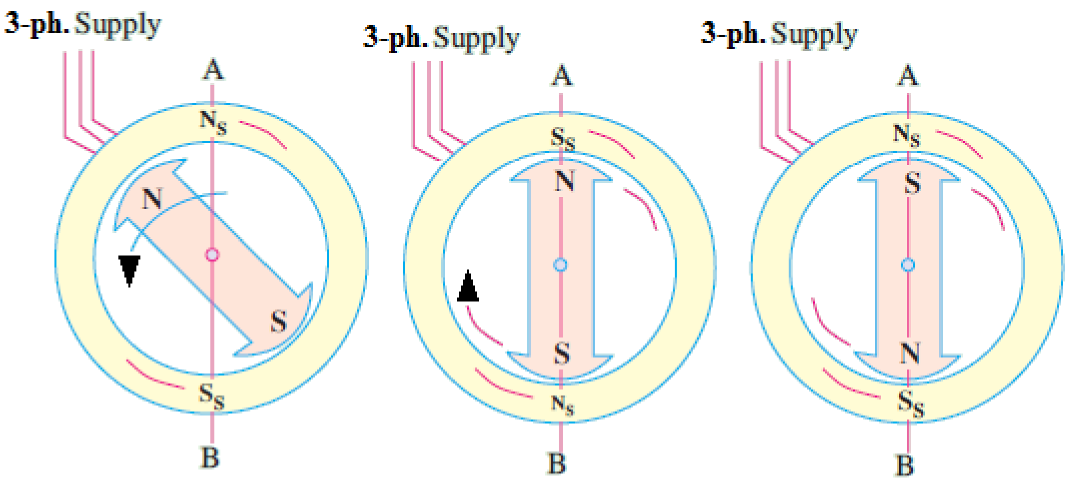


Fig. 23, Two-pole, synchronous motor



(a) (b) (c)

Fig. 24, Rotor poles with respect to stator poles

Suppose that the rotor is not stationary, but is rotating clockwise with synchronous speed. Here, again the stator and rotor poles attract each other. It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque *i.e.*, clockwise torque, as shown in Fig. 24-c.

Therefore, the rotor (which is as yet unexcited) is speeded up to synchronous or near synchronous speed by some arrangement and then excited by DC source. The moment this (near) synchronously rotating rotor is excited, it is magnetically locked into position with the stator *i.e.*, the rotor poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not at all.

However, it is important to understand that the arrangement between the stator and rotor poles is not an absolutely rigid one. As the load on the motor is increased, the rotor progressively tends to fall back in phase (but *not* in speed as in DC motors) by some angle (as shown in Fig. 25) but it still continues to run synchronously. The value of this load angle depends on the amount of load to be met by the motor. In other words, the torque developed by the motor depends on this angle, δ.

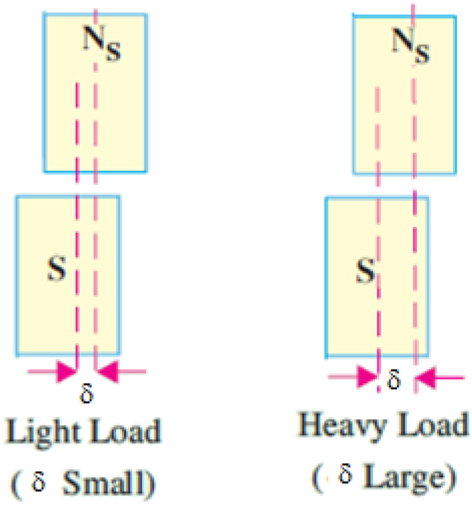


Fig. 25, Effect of increased loads on the load angle σ

8.2 Phasor Diagrams

The synchronous motor based on its equivalent circuit shown in Fig. 3-b, can be represented by the following equation:

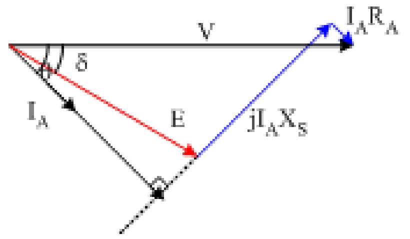
$$V = E_f + I_a(R_a + JX_S)$$

$$E_f = V - I_a(R_a + JX_S)$$

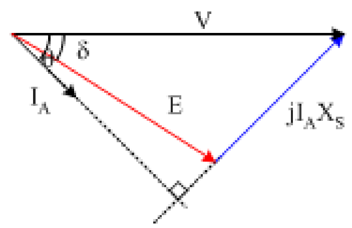
From the phasor diagram shown in Fig.26, it is clear that the synchronous motor acts as an electric load with lagging power factor and E_f < V (Under Excited motor).

From the phasor diagram shown in Fig. 27, it is clear that the synchronous motor acts as an electric load with leading power factor and E_f > V (Over Excited motor).

Also the load angle between E_f and V which called δ is always negative value.



Rₐ is considered



Rₐ is not considered

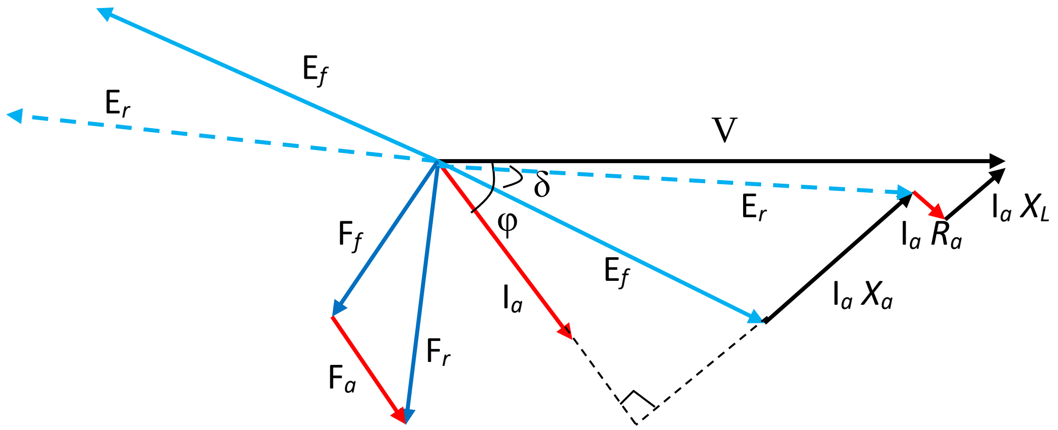
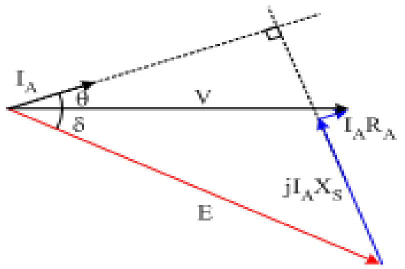
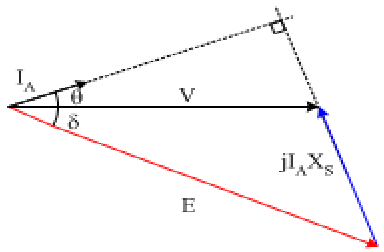


Fig. 26, Under-Excited Synchronous motor (Lagging power factor)



Rₐ is considered



Rₐ is not considered

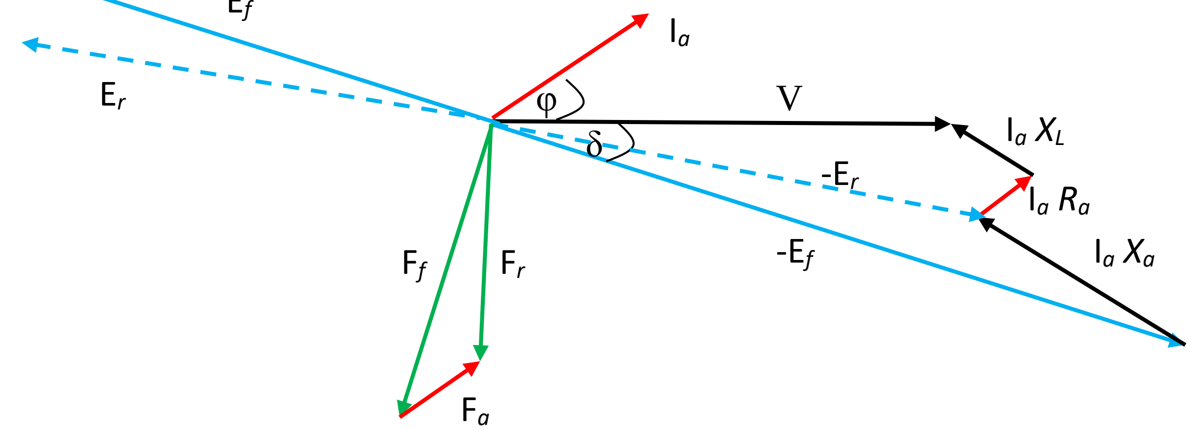
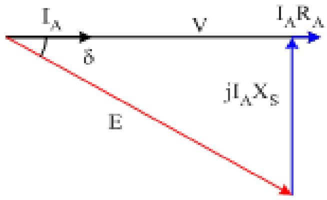
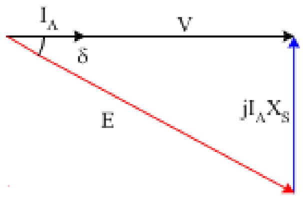


Fig. 27, Over-Excited synchronous motor (Leading power factor)





R_a is considered R_a is not considered

Fig. 28, Over-Excited synchronous motor (Unity power factor)

Summary of this point:

The load angle δ is always negative.

Also, it is seen from the MMF triangle that F_r is always leading F_f

If field excitation is such that $E_f < V$, the motor is said to be ***under-excited***. This happened at a lagging power factor as shown in Fig. 26. From this figure we notice that F_a ***support*** F_f and produce larger F_r

On the other hand, if DC field excitation is such that $E_f > V$, then motor is said to be ***over-excited*** and draws a leading current, as shown in Fig. 27. From this figure we notice that F_a ***Oppose*** F_f and produce smaller F_r

Moreover, the over-excited condition can be obtained at unity power factor as well as shown in Fig. 28.

Example (10)

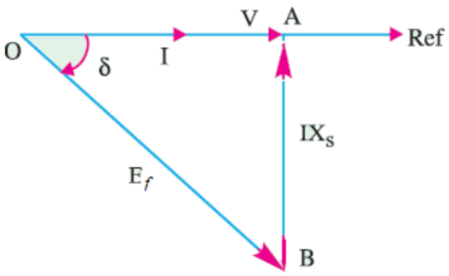
A 3-Phase synchronous motor is worked on rated values of 400 V/Ph, 32 A/Ph, and Unity p.f. X_S = 10 ohms. Neglecting the value of R_a, calculate E_f and δ.

The terminal voltage (V) = OA = 400 volt

$I_a X_S$ = AB = 32 * 10 = 320 volt

From phasr diagram, the excitation voltage $E_f = \sqrt{400^2 + 320^2} = 512.25$ volt

The load angle or torque angle (δ) = $\tan^{-1}(320/400) = 38.66$ (negative value)



8.3 Analytical Representation of Phasor Diagram

Consider the phasor diagram of 3-ph synch. motor at lagging p.f. as shown in Fig. 29-(a).

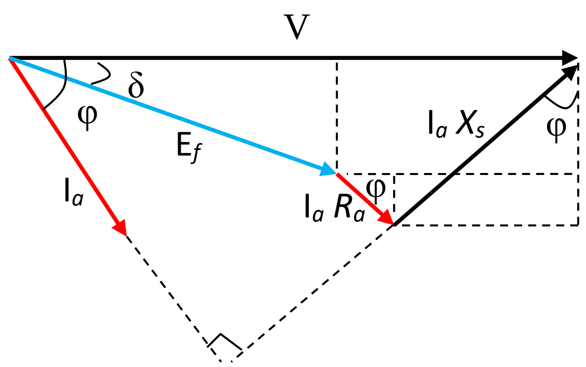


Fig. 29-(a), Synch. Motor phasor diagram at lagging p.f

We can describe this phasor diagram by two equations:

Horizontal Analysis:

$$E_f \cos(\delta) = V - I_a R_a \cos(\varphi) - \ I_a X_s \sin(\varphi) \ldots (1)$$

Vertical Analysis:

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) - I_a R_a \sin(\varphi) \ldots (2)$$

By dividing (2) over (1), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi) - I_a R_a \sin(\varphi)}{V - I_a R_a \cos(\varphi) - \ I_a X_s \sin(\varphi)}$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

Now, if R_a is neglected, the phasor diagram is shown in Fig. 29-(b).

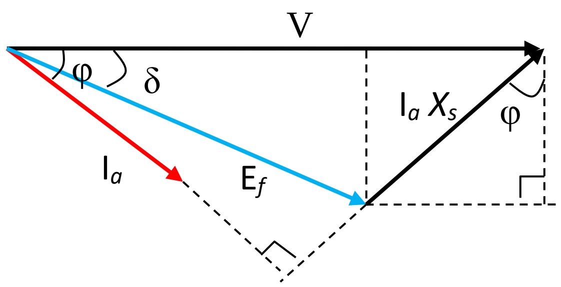


Fig. 29(b), Approximate phasor at lag p.f

Equations (1) and (2) can be rewritten by replacing R_a=0 as:

Horizontal Analysis:

$$E_f \cos(\delta) = V - \ I_a X_s \sin(\varphi) \ldots (3)$$

Vertical Analysis:

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) \ldots (4)$$

By dividing (4) over (3), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi)}{V - I_a X_s \sin(\varphi)} \ldots (5)$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

By the same way, we can describe the phasor diagram in case of leading power factor given in Fig. 30-(a) by two equations:

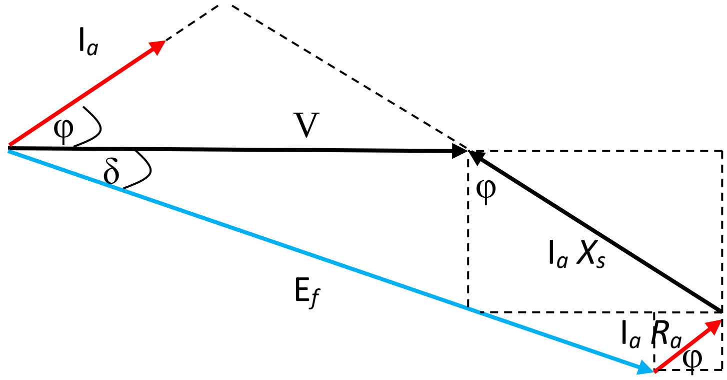


Fig. 30-(a), Synch. Motor phasor diagram at leading p.f

Horizontal Analysis:

$$E_f \cos(\delta) = V + I_a X_s \sin(\varphi) - I_a R_a \cos(\varphi) \ldots (6)$$

Vertical Analysis:

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) + I_a R_a \sin(\varphi) \ldots (7)$$

By dividing (7) over (6), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi) + I_a R_a \sin(\varphi)}{V + I_a X_s \sin(\varphi) - I_a R_a \cos(\varphi)}$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

Now, if R_a is neglected, the phasor diagram is shown in Fig. 30(b).

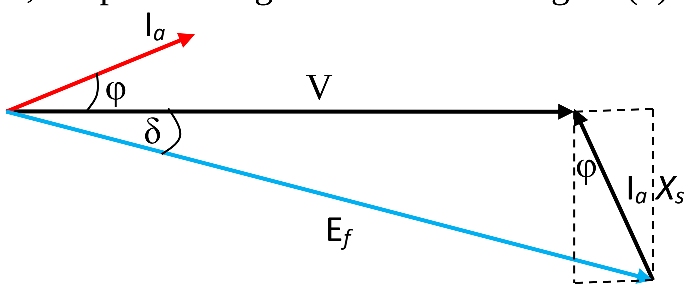


Fig. 30-b, Approximate phasor at lead p.f

Equations (6) and (7) can be rewritten by replacing R_a=0 as:

Horizontal Analysis:

$$E_f \cos(\delta) = V + I_a X_s \sin(\varphi) \ldots (8)$$

Vertical Analysis:

$$E_f \sin(\delta) = I_a X_s \cos(\varphi) \ldots (9)$$

By dividing (9) over (8), we obtain:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi)}{V + I_a X_s \sin(\varphi)}$$

Once the angle (δ) is known, we can obtain the excitation voltage E_f.

From the explained phasor diagrams given in Figs 26 to 30, we notice that E_f is always behind V, this means the power angle (δ) is always negative, and this is the remarkable notice on the phasor diagram of synchronous motors.

9. Power Flow within a Synchronous Machines

9.1 Synchronous Generators power

When a synchronous machine is operated as a generator, a prime mover is required to drive the generator as shown in Fig. 31.

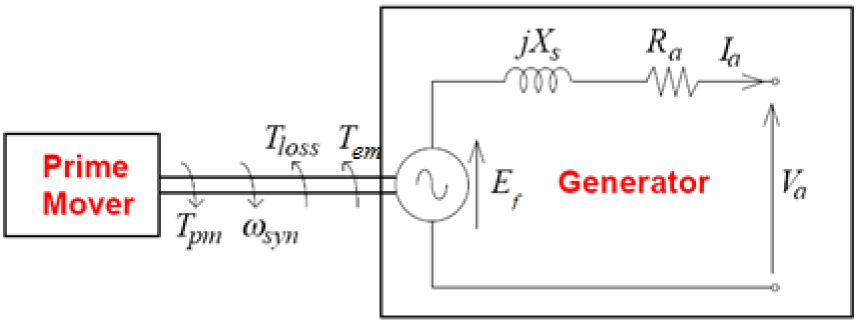


Fig. 31, Synchronous generator configuration

In steady state, the mechanical torque of the prime mover (T_{pm}) should balance with the electromagnetic torque (T_{em}) produced by the generator and the mechanical loss torque (T_{Loss}) due to friction and windage, as:

$$T_{pm} = T_{em} + T_{loss}$$

Multiplying the synchronous speed to both sides of the torque equation, we have the power balance equation as:

$$P_{pm} = P_{em} + P_{loss}$$

Where P_{pm} is Prime Mover power or the total mechanical power input to the generator or called the Gross Mechanical power = $\omega_{syn}T_{pm}$

P_{loss} is the mechanical power loss or the friction and windage loss = $\omega_{syn}T_{loss}$

P_{em} is the electromagnetic power that converted to electrical power = $\omega_{syn}T_{em}$

$P_{em} = \omega_{syn}T_{em} = 3\ E_f\ I_a \cos(\phi + \delta)$ in case of Lagging P.F.

$P_{em} = \omega_{syn}T_{em} = 3\ E_f\ I_a \cos(\phi - \delta)$ in case of Leading P.F.

Taking the iron loss power (P_{iron}) and the armature cupper loss (P_{cu}), we can get the electrical output power (P_{out}). The power flow of alternators is shown in Fig. 32.

$$P_{out} = 3\ V_a\ I_a \cos(\phi)$$

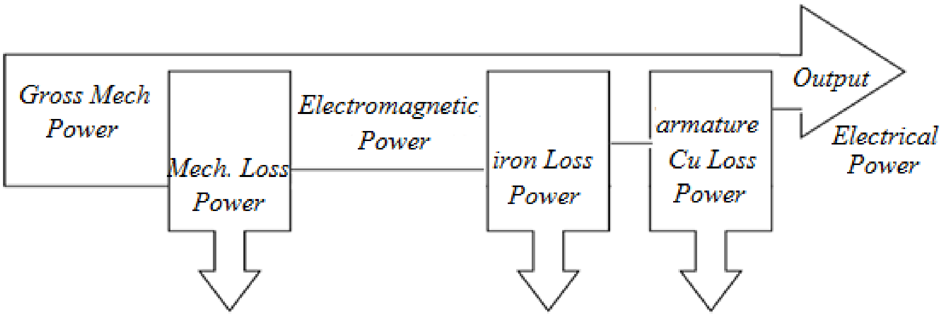


Fig. 32, Power flow in a synchronous generator

As we know, in case of generator

$$E_f = V + I_a(R_a + JX_S)$$

The value of X_S is much bigger compared to the value of R_a, so R_a can be neglected.

$$E_f = V + I_a(JX_S)$$

That can be represented by the phasor diagram shown in Fig. 33.

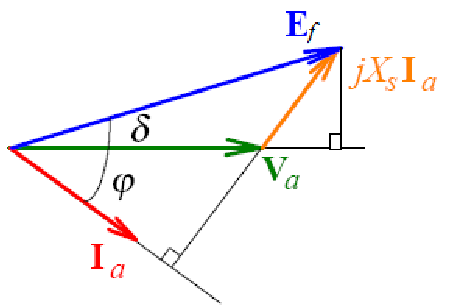


Fig. 33, phasor diagram for synchronous generator when neglecting R_a

From phasor diagram (Fig. 13), $E_f \sin(\delta) = I_aX_S \cos(\phi) \rightarrow I_a \cos(\phi) = E_f \sin(\delta) / X_S$

Therefore, $P_{out} = 3\ V_a\ I_a \cos(\phi)$

$$P_{out} = \frac{3\ V_a\ E_f}{X_S} \sin(\delta)$$

If R_a is considered, the expression of the output power P_{out} will be

$$P_{out} = \frac{3\ V_a\ E_f}{Z_S} \cos(\theta - \delta) - \frac{3\ {E_f}^2}{Z_S} \cos(\theta)$$

Where θ the impedance angle = $\tan^{-1}(X_S/R_a)$

If the iron loss power (P_{iron}) and the armature cupper loss (P_{cu}) are neglected, therefore $P_{out} = P_{em}$ and the electromagnetic torque (T) can be obtained as

$$T = \frac{3\ V_a\ E_f}{\omega_{syn} X_S} \sin(\delta)$$

When the stator winding resistance is ignored, δ can be regarded as the angle between the rotor and stator rotating magnetic fields. The electromagnetic torque of a synchronous machine is proportional to the sine function of the load angle, as plotted in Fig. 34, where the curve in the third quadrant is for the situation when the machine is operated as a motor, where the electromagnetic torque is negative because the armature current direction is reversed.

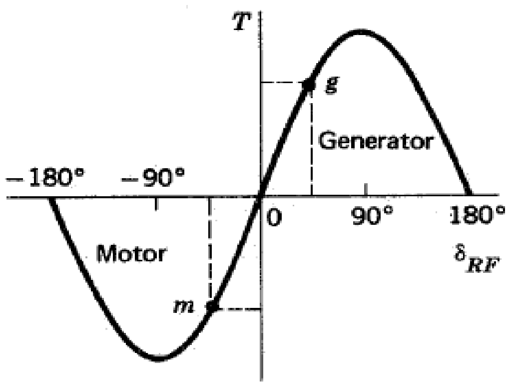


Fig. 34, T-δ relationship

9.2 Synchronous Motor power

When a synchronous machine is operated as a motor to drive a mechanical load as shown in Fig. 35, in steady state, the mechanical torque of the motor should balance the load torque and the mechanical loss torque due to friction and windage, that is

$$T_{em} = T_{Load} + T_{loss}$$

Multiplying the synchronous speed to both sides of the torque equation, we have the power balance equation as

$$P_{em} = P_{Load} + P_{loss}$$

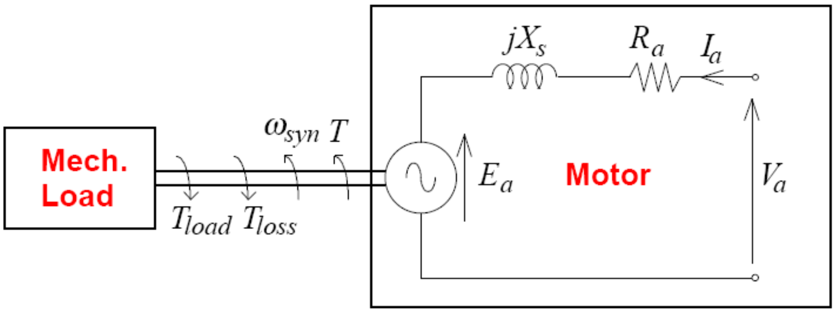


Fig. 35, Synchronous motor configuration

Similar to the case of a generator, the electromagnetic power is the amount of power being converted from the electrical into the mechanical power. That is

$P_{em} = \omega_{syn}T = 3\ E_f\ I_a \cos(\delta - \phi)$ in case of Lagging P.F.

$P_{em} = \omega_{syn}T = 3\ E_f\ I_a \cos(\delta + \phi)$ in case of Leading P.F.

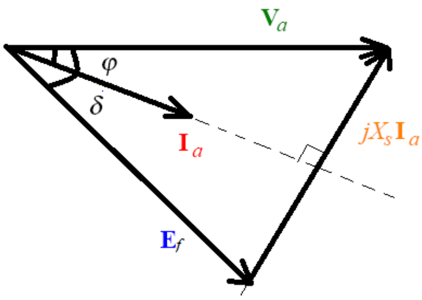


Fig. 36, phasor diagram for synchronous motor when R_a is neglected

From the phasor diagram shown in Fig. 36, $E_f \cos(\delta - \phi) = V_a \cos(\phi)$

Also, $E_f \sin(\delta) = I_a X_S \cos(\phi)$

Therefore, the electromagnetic power can be expressed as

$$P_{em} = \frac{3\ V_a\ E_f}{X_S}\sin(\delta)$$

$$T = \frac{3\ V_a\ E_f}{\omega_{syn} X_S}\sin(\delta)$$

The T-δ relationship for the synchronous motor is shown in Fig. 34 (Third Quadrant). The input electrical power to the motor is $P_{in} = 3\ V_a\ I_a \cos(\phi)$

Taking the armature copper loss (P_{cu}) and the iron loss (P_{iron}) to get the electromagnetic power P_{em} as shown in the power flow shown in Fig. 37.

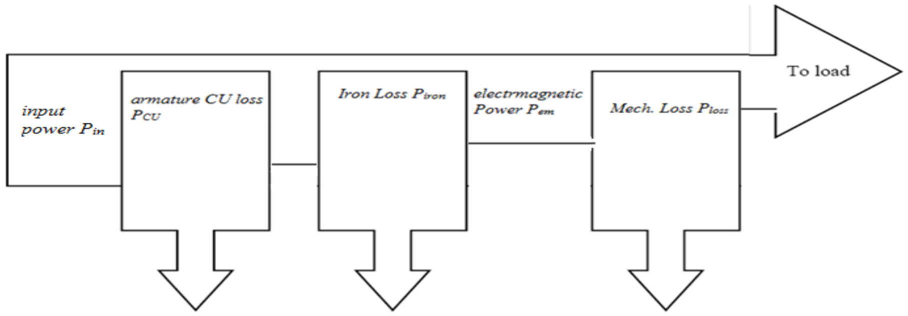


Fig. 37, power flow in synchronous motor

10.1 Different Torques of a Synchronous Motor

Various torques associated with a synchronous motor are as follows:

1. Starting torque (developed by the motor at starting)

2. Running torque (developed by the motor at the running conditions)

3. Pull-in torque (A synchronous motor is started as induction motor till it runs at 2 to 5% below the synchronous speed. Afterwards, excitation is switched on and the rotor pulls into step with the synchronously rotating stator field. The amount of torque at which the motor will pull into step is called the pull-in torque.

4. Pull-out torque (The maximum torque that the motor can develop without pulling out of synchronism)

To get the Pull-out Torque, we get the maximum power then divide by the synchronous speed.

If R_a is neglected; the max. power obtained at δ = 90 as shown in Fig. 38

$$(P_{em})_{max} = \frac{3\,V_a\,E_f}{X_S}\sin(90) = \frac{3\,V_a\,E_f}{X_S}$$

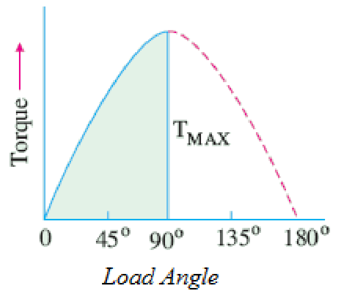


Fig. 38, Maximum (Pull-out) torque

If R_a is not neglected, therefore to get the maximum power is at $dP_{em}/d\delta = 0$

$$\frac{dP_{em}}{d\delta} = -\frac{3\,V_a\,E_f}{Z_S}\sin(\theta-\delta) = 0$$

This occurred at $\delta = \theta$,

Then the maximum power can be expressed as:

$$(P_{em})_{max} = \frac{3\,V_a\,E_f}{Z_S} - \frac{3\,{E_f}^2 R_a}{{Z_S}^2}$$

Example (11):

A 75-kW, 3-φ, Y-connected, 50-Hz,4-pole, 440-V cylindrical rotor synchronous motor operates at rated condition with 0.8 p.f. leading. The motor efficiency is 95% and X_S = 2.5 Ω. *Calculate* **(*i*)** *input power* **(*ii*)** *armature current* **(*iii*)** *back e.m.f.* **(*iv*)** *power angle* **(*v*)** *electromagnetic power* **(*vi*)** *maximum or pull-out torque of the motor.*

P_{out} = 75000 W, N_s=60×50/2=1500 rpm, ω_s=2π×1500/60=157.08 rad/s

i) the motor efficiency $\eta = P_{out}/P_{in}$

$$0.95 = \frac{75000}{P_{in}} \rightarrow P_{in} = \frac{75000}{0.95} = 78947.368\,W$$

ii) $P_{in} = \sqrt{3}\ V\ I_a \cos(\varphi)$

$$I_a = \frac{78947.368}{\sqrt{3}\times 440\times 0.8} = 129.489\,A$$

iv) Based on Horizontal and vertical analysis for leading power factor motor:

$$\tan(\delta) = \frac{I_a X_s \cos(\varphi)}{V + I_a X_s \sin(\varphi)} = \frac{129.489\times 2.5\times 0.8}{\frac{440}{\sqrt{3}} + 129.489\times 2.5\times \sin(36.87)} = 0.5777$$

$$\delta = tan^{-1}(0.5777) = 30°$$

iii) From vertical analysis

$$E_f \sin(\delta) = I_a X_s \cos(\varphi)$$

Then

$$E_f = \frac{I_a X_s \cos(\varphi)}{\sin(\delta)} = \frac{129.489\times 2.5\times 0.8}{\sin(30)} = 517.956\,V$$

v) the electromagnetic power P_{em} is given as

$$P_{em} = 3\,E_f\,\mathrm{I_a}\cos(\delta+\varphi) = 3\times 517.956\times 129.489\cos(30+36.87)$$
$$= 79039.1\,W$$

vi) the pull out torque occurs at δ=90

$$T_{pull-out} = \frac{3\,V_a\,E_f}{\omega_S X_S}\sin(90) = \frac{3\times\frac{440}{\sqrt{3}}\times 517.956}{157.08\times 2.5} = 1005.183\,N.m$$

10.2 Synchronous Machines Run at constant power

Assume that a synchronous motor is driving a constant torque load. The active power converted by the machine is constant, no matter what the value of the field current is, since the motor speed is a constant. Thus,

$$T = \frac{3\,V_a\,E_f}{\omega_{syn} X_S}\sin(\delta) = \frac{3\,V_a\,\mathrm{I_a}\cos(\phi)}{\omega_{syn}} = constant$$

$$E_f\sin(\delta) = I_a\cos(\phi) = constant$$

Using the phasor diagram shown in Fig. 39, we analyze the variation of the power factor angle of a synchronous motor when the field excitation is varied. Both E_f and I_a can only be able to vary along the horizontal and the vertical dotted lines.

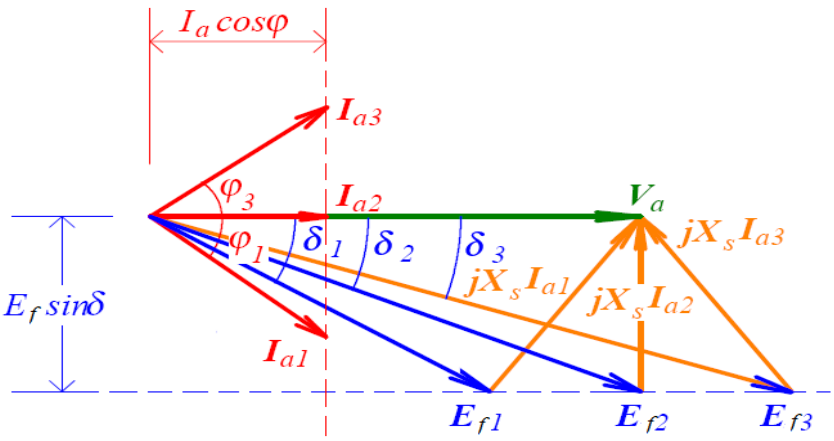


Fig. 39, Synchronous Motor driving constant load

For a small rotor field current **(under excited)**, the induced *emf* shown by the phasor $\mathbf{E}_{f1}$. This yields a lagging power factor angle ϕ_1.

As the excitation current increases **(over excited)**, the lagging power factor angle is reduced. At a certain rotor current, the induced *emf* given by phasor $\mathbf{E}_{f2}$ is perpendicular to the terminal voltage phasor, and hence the stator current phasor is aligned with the terminal voltage, that is a zero power factor angle $\phi_2 = 0$.

When the rotor current further increases **(over excited)**, the stator current leads the terminal voltage, or a leading power factor angle ϕ_3.

In practice, because of this feature, synchronous motors are often run at no active load as ***synchronous condensers*** for the purpose of power factor correction. The diagram shown in Fig. 40, illustrates schematically the power factor compensation for an inductive load, which is common for factories using large induction motor drives, using a synchronous condenser. By controlling the rotor excitation current such that the synchronous condenser draws a line current of leading phase angle, whose imaginary component cancels that of the load current, the total line current would have a minimum imaginary component. Therefore, the overall power factor of the inductive load and the synchronous condenser would be close to one and the magnitude of the overall line current would be the minimum.

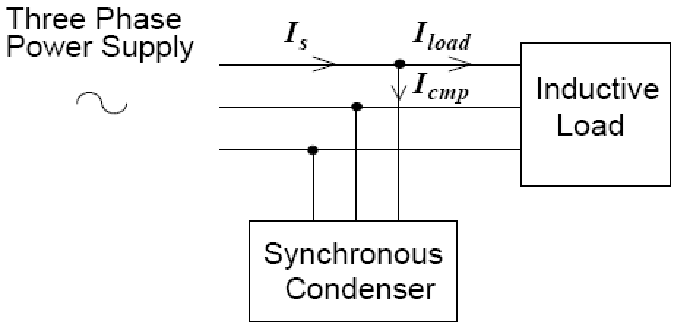
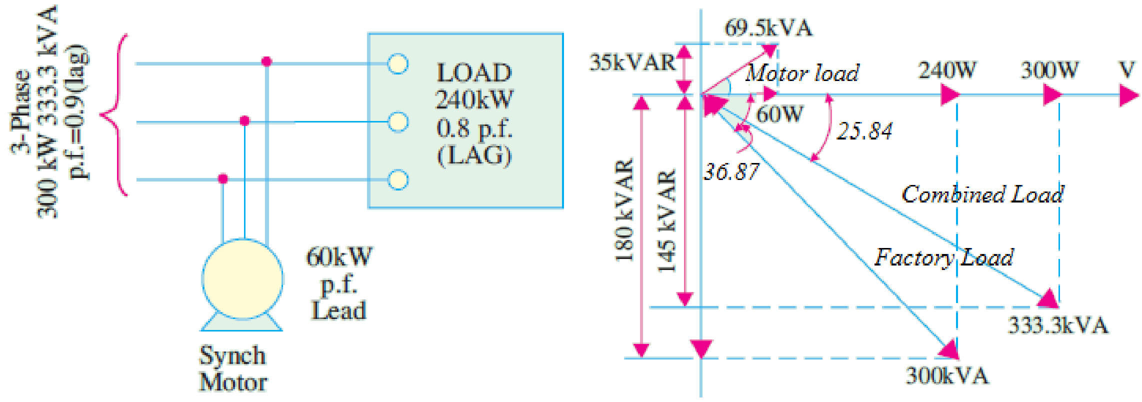


Fig. 40, Over-excited synchronous motor for P.F. improvement

It can also be seen that only when the power factor is unity or the stator current is aligned with the terminal voltage, the magnitude of the stator current is minimum.

Example (12):

A synchronous motor absorbing 60 kW is connected in parallel with a factory load of 240 kW having a lagging p.f. of 0.8. If the combined load has a p.f. of 0.9, what is the value of the leading kVAR supplied by the motor and at what p.f. is it working ?



P_{Motor} = 60 kW

$P_{Factory}$ = 240 kW, $Q_{Factory}$ = 240000×tan(36.87)=180 kVAR

$P_{combined}$ = 240 + 60 = 300 kW

since the P.F. of the combined load is 0.9 lagging, then

$Q_{combined}$ = 300000×tan(25.84) = 145 kVAR (Lag)

Therefore, Q_{motor} = 180 - 145.297 = 35 kVAR (Lead)

Motor angle = $\tan^{-1}$ 35/60 = 30.3°

Motor P.F. = cos (30.26) = 0.863 lead

When the motor power is kept constant, then by plotting the magnitude of the stator current against the rotor excitation current at different loading conditions, a family of "V" curves can be obtained as shown in Fig. 41.

It is shown that a larger rotor field current is required for a larger active load to operate at unity power factor.

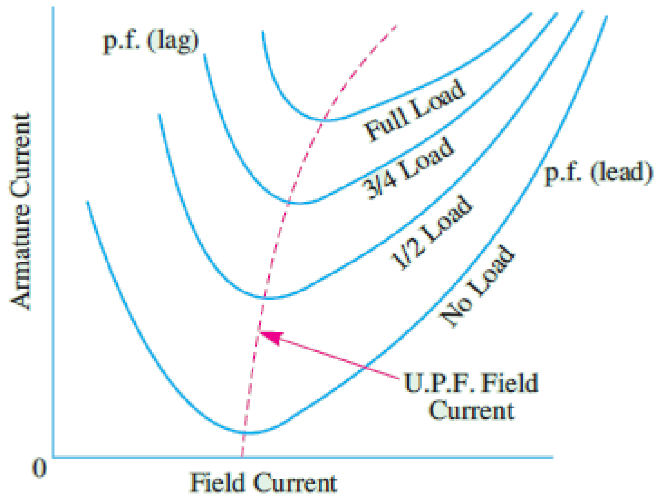


Fig. 41, V-curves of synchronous motor

When the generator power is kept constant, then by plotting the magnitude of the stator current against the rotor excitation current at different loading conditions, a family of "V" curves can be obtained as shown in Fig. 42.

It is shown that a larger rotor field current is required for a larger active load to operate at unity power factor.

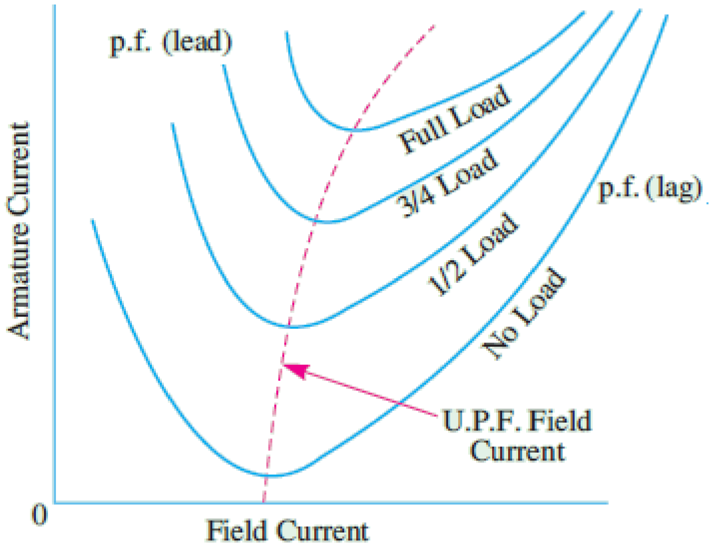


Fig. 42, V-curves of synchronous generator

Example (13)

A 3-phase, 400-V, Y-connected, 4-pole, cylindrical synchronous generator with synchronous reactance of 10Ω and negligible armature resistance. For the requirements of constant power operation, the value of "$E_f \sin(\sigma)$" is kept constant at 57.6 V. When supplying a load with 0.8 p.f. lag, calculate:

a) The excitation voltage (E_f) and the load angle (σ),

b) The armature current (I_a).

If the generator is running under constant power condition and the load p.f. is increased to unity. Calculate the new values of:

c) The excitation voltage (E_f) and the load angle (σ),

d) The armature current (I_a).

Graphical Solution

Assuming the voltage scale is each 10 V is represented by 1 cm

$V_a = 400/\sqrt{3} = 230.94$ V = 23.1 cm and represented by line OA

The current direction is OB at angle $\phi = 36.9$ lag

Draw the line CD parallel to V_a at a distance E_f sin(σ) is constant = 57.6 V = 5.8 cm

Draw a line ⊥ OB and intersect with CD in point G

Measure OG that represents the excitation voltage E_f = 28 cm = 280 V ######

Measure the angle AOG that represents the load angle (σ) = 12 ####

Measure AG that represents $I_a X_S$ = 7.2 cm = 72 V

$I_a = AG/X_S$ = 72/10 = 7.2 A ####

<u>Now, the power factor is increased to 1.0</u>

Measure OH that represents the excitation voltage E_f = 23.8 cm = 238 V ######

Measure the angle AOH that represents the load angle (σ) = 14 ####

The drop $I_a X_S$ is represented by AH = 5.8 cm = 57.6 V

$I_a = AH/X_S$ = 57.6/10 = 5.76 A ######

<u>Analytical Solution</u>

From phasor diagram

$E_f \sin(\sigma) = I_a X_S \cos(\phi)$ ---------------------------------(1)

$E_f \cos(\sigma) = V_a + I_a X_S \sin(\phi)$ --------------------------(2)

From (1)

$$I_a = \frac{E_f \sin(\sigma)}{X_S \cos(\phi)} = \frac{57.6}{10 \times 0.8} = 7.2\ A$$

From (2)

$$E_f \cos(\sigma) = 230.94 + 72 \sin(36.87) = 274.14$$

$$\frac{E_f \sin(\sigma)}{E_f \cos(\sigma)} = \tan(\sigma) = \frac{57.6}{274.14} = 0.210111$$

$$\sigma = 11.87°$$

$$E_f \sin(11.87) = 57.6$$

$$E_f = \frac{57.6}{\sin(11.87)} = 280.126\ V$$

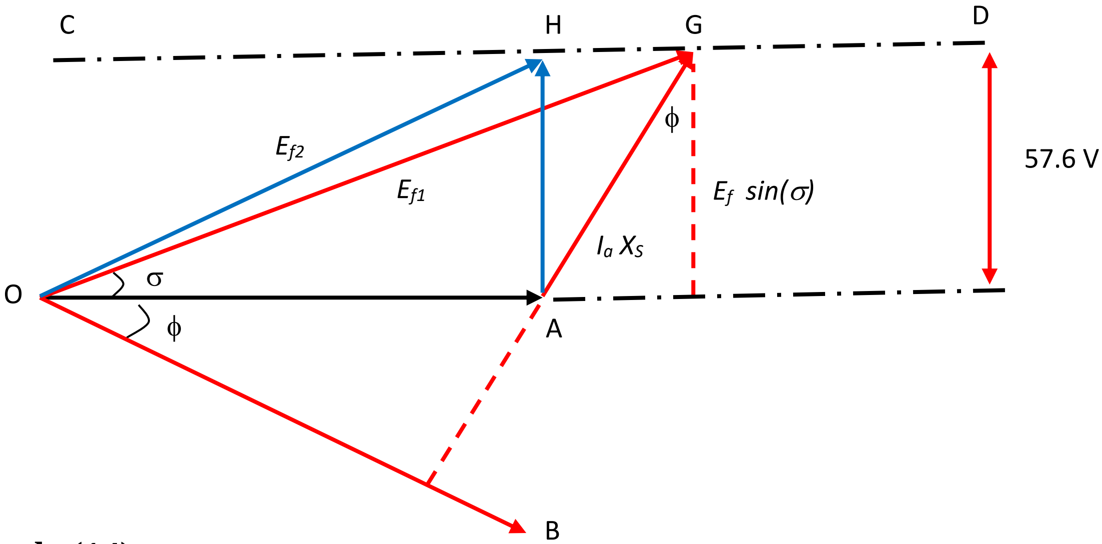
<u>Now, the power factor is increased to 1.0</u>

$I_a X_S = E_f \sin(\sigma)$ = 57.6 V

I_a = 57.6 / 10 = 5.76 A

$$E_f = \sqrt{230.94^2 + 57.6^2} = 238.015\ V$$

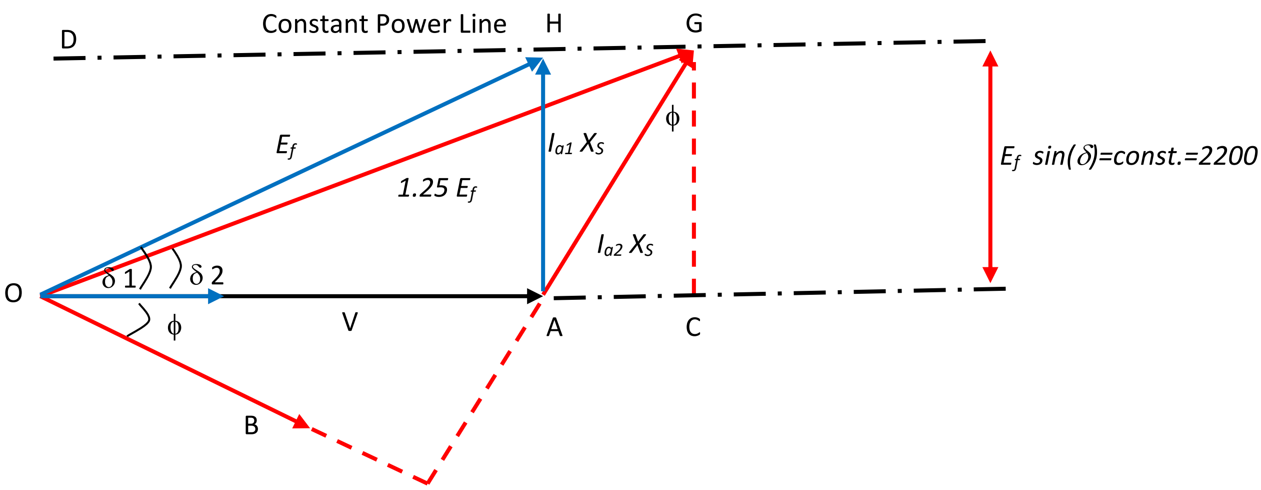
$$\sigma = tan^{-1}\frac{57.6}{230.94} = 14°$$



Example (14):

A 3-phase, Y-connected turbo-alternator, having a synchronous reactance of 10 Ω per phase and negligible armature resistance, has an armature current of 220 A at unity p.f. The supply voltage is constant at 11 kV at constant frequency. If the steam admission is unchanged and the e.m.f. raised by 25%, determine the current and power factor.

If the higher value of excitation is maintained and the steam supply is slowly increased, at what power output will the alternator pulls out of synchronism?



Since unity power factor, the drop $I_{a1} X_s$ is ⊥ V giving the triangle OAH

$$E_f = \sqrt{\left(\frac{11000}{\sqrt{3}}\right)^2 + (220 \times 10)^2} = 6721.11\,V$$

$$\delta_1 = tan^{-1}\frac{2200}{6350.853} = 19.11°$$

$$E_f \sin(\delta 1) = 6721.11 \times \sin(19.11) = 2200$$

Since steam admission is unchanged, this mean the power is held constant and this is represented by the constant power line DG

The induced emf increased by 25% = E_{f2}= 1.25×6721.11=8401.388 V

E_{f2} sin(δ2)=2200

$$\delta_2 = sin^{-1}\left(\frac{2200}{8401.388}\right) = 15.181°$$

OC = E_{f2} cos(δ_2) = 8401.388×cos(15.181) = 8108.225

AC = 8108.225 - (11000/√3) = 1757.3725

$$\varphi = tan^{-1}\frac{1757.3725}{2200} = 38.618°$$

P.F. = cos(38.618) = 0.7813 Lagging

I_{a2} X_S sin(φ) = 1757.3725

I_{a2} = 281.574 A

Since the steam is increased until *pulls out of synchronism*, this mean max. power

$$P_{max} = \frac{3\,V_a\,E_f}{X_S}\sin(90) = \frac{3 \times \frac{11000}{\sqrt{3}} \times 8401.388}{10} = 16.006794\,MW$$

Example (15)

A 3-phase, 11.0 kV, 4.2 MW, Y-connected turbo-alternator, having a synchronous reactance of 10 Ω per phase and an armature resistance of 1.0 Ω per phase, is working at full load at unity p.f. If the steam admission is unchanged and the e.m.f. raised by 35%, determine:

a) the armature current and power factor.
b) The output power

If the higher value of excitation is maintained and the steam supply is slowly increased,

c) At what power output will the alternator pulls out of synchronism?
d) Calculate the armature current and power factor under this condition.

At full load at unity p.f.

$$I_a = \frac{4.2 \times 10^6}{\sqrt{3} \times 11000 \times 1} = 220.44283\ A$$

$$E_f = \sqrt{\left(\frac{11000}{\sqrt{3}} + 220.44283\right)^2 + (220.44283 \times 10)^2} = 6931.19272\ V$$

$$\tan(\delta) = \frac{220.44283 \times 10}{\frac{11000}{\sqrt{3}} + 220.44283} = 0.3355$$

$$\delta = 18.54471°$$

Now, E_f increased by 35% and the power remains constant:

$$E_f = 1.35 \times 6931.19272 = 9357.1102\ V$$

Since the power is constant:

$$E_{f1}\sin(\delta_1) = E_{f2}\sin(\delta_2)$$

$$6931.19272\sin(18.54471) = 6931.19272\ \sin(\delta_2)$$

$$\delta_2 = 13.627°$$

At this condition:

$$E_f\cos(\delta) = V + I_aR_a\cos(\varphi) + I_aX_s\sin(\varphi) \qquad (1)$$

$$E_f\sin(\delta) = I_aX_s\cos(\varphi) - I_aR_a\sin(\varphi) \qquad (2)$$

From (2)

$$I_a\cos(\varphi) = \frac{E_f\sin(\delta)+I_aR_a\sin(\varphi)}{X_s} \qquad (3)$$

Substituting in (1)

$$E_f\cos(\delta) = V + R_a\frac{E_f\sin(\delta) + I_aR_a\sin(\varphi)}{X_s} + I_aX_s\sin(\varphi)$$

$$E_f\cos(\delta) = V + \frac{R_aE_f\sin(\delta)}{X_s} + I_a\sin(\varphi)\left[X_s + \frac{{R_a}^2}{X_s}\right]$$

From the above equation

$$I_a\sin(\varphi) = 249.74275 \qquad (4)$$

Substituting with this value in (3)

$$I_a\cos(\varphi) = 245.4279 \qquad (5)$$

Dividing (5) over (4) and taking inverse tan:

$$\varphi = 45.5 \ \rightarrow p.f = 0.701\ lag$$

From (4) $I_a = 350.152\ A$

The output power is:

$$P_{out} = \frac{3\ V_a\ E_f}{Z_S}\cos(\theta - \delta) - \frac{3\ {E_f}^2}{Z_S}\cos(\theta)$$

Where θ the impedance angle = $\tan^{-1}(X_S/R_a)$

$$Z_S = 10.04988, \qquad \theta = 84.2894°$$

$$P_{out} = \frac{3 \times \frac{11000}{\sqrt{3}} \times 9357.1102}{10.04988}\cos(84.2894 - 13.627)$$
$$- \frac{3 \times 9357.1102^2}{10.04988}\cos(84.28)$$

$$P_{out} = 5874048.7 - 2600660.733 = 3273387.967\ W$$

For maximum power, $dP_{out}/d\delta = 0$

$$\frac{dP_{out}}{d\delta} = -\frac{3\ V_a\ E_f}{Z_S}\sin(\theta - \delta) = 0$$

This occurred at δ = θ,

$$\delta = 84.2894°$$

Then the maximum power can be expressed as:

$$(P_{out})_{max} = \frac{3\ V_a\ E_f}{Z_S} - \frac{3\ {E_f}^2 R_a}{{Z_S}^2}$$

$$(P_{out})_{max} = \frac{3 \times \frac{11000}{\sqrt{3}} \times 9357.1102}{10.04988} - \frac{3 \times 9357.1102^2}{10.04988^2}$$

$$(P_{out})_{max} = 17739206.15 - 2600656.485 = 15138549.66\ W$$

At this condition:

$$E_f\cos(\delta) = V + I_a R_a\cos(\varphi) - I_a X_s\sin(\varphi) \qquad (1)$$

$$E_f\sin(\delta) = I_a X_s\cos(\varphi) + I_a R_a\sin(\varphi) \qquad (2)$$

From (2)

$$I_a\cos(\varphi) = \frac{E_f\sin(\delta) - I_a R_a\sin(\varphi)}{X_s} \qquad (3)$$

Substituting in (1)

$$E_f\cos(\delta) = V + R_a\frac{E_f\sin(\delta) - I_aR_a\sin(\varphi)}{X_s} - \ I_aX_s\sin(\varphi)$$

$$E_f\cos(\delta) = V + \frac{R_aE_f\sin(\delta)}{X_s} - \ I_a\sin(\varphi)\left[X_s + \frac{{R_a}^2}{X_s}\right]$$

From the above equation

$$I_a\sin(\varphi) = 628.7972 \qquad (4)$$

Substituting with this value in (3)

$$I_a\cos(\varphi) = 868.1875 \qquad (5)$$

Dividing (5) over (4) and taking inverse tan:

$$\varphi = 54.085 \ \ \rightarrow p.f. = 0.5866 \ \ (lead)$$

From (4)

$$I_a = 776.3949\ A$$

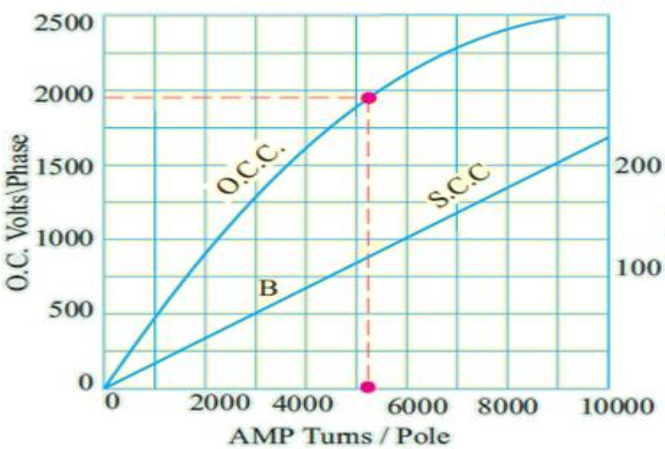
Example (16):

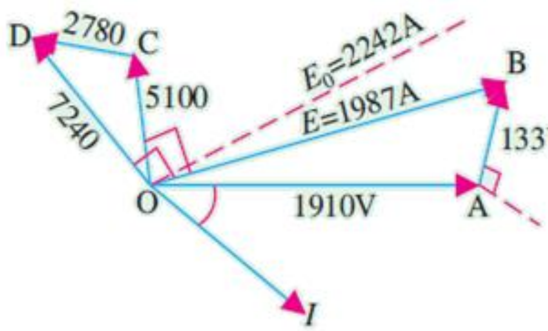
A 600-kVA, 3300-V, 8-pole, 3-phase, 50-Hz alternator has following characteristic:

Amp-turns/pole	: 4000	5000	7000	10,000
Terminal E.M.F.	: 2850	3400	3850	4400

There are 200 conductors in series per phase.

Find the short-circuit characteristic, the field ampere-turns for full-load 0.8 p.f. (lagging) and the voltage regulation. Given that the inductive drop at full-load is 7% and that the equivalent armature reaction in amp-turns per pole at Z.P.F = 1.06 × ampere-conductors per phase per pole. O.C. terminal voltages are first converted into phase voltages and plotted against field amp turns, as shown in Fig.





Full-load current

$$= \frac{600,000}{\sqrt{3}\times 3300} = 105 \text{ A}$$

Demagnetising amp-turns per pole per phase for full-load at zero p.f. = 1.06×105×200/8

F_a = 2780 A.t/pole

Phase voltage = 3300/√3 = 1910 V

Leakage reactance drop = 0.07×1910 = 133 V

From phasor diagram, *OA* represents phase voltage, *AB* which represent I_aX_L is drawn ⊥ current *OI*, *OB* is the resultant voltage $E_r = 1987\ V$

From O.C.C., we find that 1987 V correspond to 5100 field amp-turns that represent F_r. Hence, *OC* = 5100 is drawn ⊥ to OB.

CD represent F_a = 2780 is // to the current *OI*. Hence, *OD* that represent F_f = 7240 (approx). From O.C.C. it is found that this corresponds to an O.C. voltage of 2242 volt. Hence, when load is thrown off, the voltage will rise to 2242 V.

$$\%\ \text{regn.} = \frac{2242-1910}{1910}\times 100$$

$$= 17.6\ \%$$

We have found that field amp-turns for balancing armature reaction only are 2,780. To this should be added field amp-turns required for balancing the leakage reactance voltage drop of 133 V. Field amp-turns corresponding to 133 volt on O.C. are 300 approximately.

∴ Short-circuit field amp-turns = 2780 + 300 = 3080

Hence, we get a point *B* on S.C.C. *i.e*. (3080, 105) and the other point is the origin. So S.C.C. (which is a straight line).

Sheet (2) Cylindrical Synchronous Machines

1- In a 50-kVA, Y-connected, 440-V, 3-phase, 50-Hz alternator, the effective armature resistance is 0.25 Ω per phase. The synchronous reactance is 3.2 Ω per phase and leakage reactance is 0.5 Ω per phase. Determine at rated load and unity power factor :

(a) Line value of the internal resultant e.m.f. E_r

(b) Line value of the no-load e.m.f. E_f

(c) Percentage regulation on full-load

(d) Armature reactance.

[471 V, 592V, 34.65%, 2.7Ω]

2- A 1000 kVA, 3300-V, 3-phase, Y-connected alternator delivers full-load current at rated voltage at 0.80 p.f. Lagging. The resistance and synchronous reactance of the machine per phase are 0.5 Ω and 5 Ω respectively. Estimate the terminal voltage for the same excitation and same load current at 0.80 p.f. leading.

[2925 V]

3- Find the synchronous impedance and reactance of an alternator in which a given field current produces an armature current of 200 A on short-circuit and a generated e.m.f. of 50 V on open-circuit. The armature resistance is 0.1 Ω. To what induced voltage must the alternator be excited if it is to deliver a load of 100 A at a p.f. of 0.8 lagging, with a terminal voltage of 200V.

[0.25Ω, 0.23Ω, 222 V]

4- From the following test results, determine the voltage regulation of a 2000-V(phase), Y-connected 3-phase alternator delivering a current of 100 A at

(i) unity p.f. (ii) 0.8 leading p.f. and (iii) 0.71 lagging p.f.

Test results : Full-load current of 100 A is produced on short-circuit by a field excitation of 2.5A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is 0.8Ω.

[7%, -9%, 21.6%]

5- A 100-kVA, 3000-V, 50-Hz 3-phase star-connected alternator has effective armature resistance of 0.2 Ω. The field current of 40 A produces short-circuit current of 200 A and an open-circuit emf of 1040 V (line value). Calculate the full-load voltage regulation at 0.8 p.f. lagging and 0.8 p.f. leading. Draw phasor diagrams.

[2.2%, -1.8%]

6- A 3-phase, star-connected alternator is rated at 1600 kVA, 13,500 V. The armature resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at 0.8 leading power factor.

[-14.11%]

7- A 3-phase, 10-kVA, 400-V, 50-Hz, Y-connected alternator supplies the rated load at 0.8 p.f. lag. If the resistance is 0.5 Ω and syn. reactance is 10 Ω, find the power angle and voltage regulation.

[18.39°, 48%]

8- The following test results are obtained from a 3-phase, 6,000-kVA, 6,600 V, Y-connected, 2-pole, 50-Hz turbo-alternator:

With a field current of 125 A, the OC voltage is 8,000 V at the rated speed; with the same field current and rated speed, the SC current is 800 A. At the rated full-load, the resistance drop is 3%. Find the regulation of the alternator on full-load and at a power factor of 0.8 lagging. [62.2%]

9- A 3-phase, Y-connected, 3300 V, synchronous generator which supplies 1000 kW at 80 percent power factor lag, if its OCC is given by the following data:

Field Current (A)	80	96	118
OCC line voltage	3300	3600	3900

the stator winding has a per phase resistance of 0.15 ohm and a leakage reactance of 1.2 ohm. There are 16 poles, each wound with 108 turns. The armature winding is arranged in 144 slots, 5 conductors per slot, with full-pitched coils.

a) Calculate the armature MMF in AT/Pole

b) Estimate the field current (I_f) for this loading condition

Answer (F_a = 4250 AT/Pole & I_f = 126 A)

10- Calculate the saturated synchronous reactance (in Ω/phase and per unit) of a 85-kVA synchronous machine which achieves its rated open-circuit voltage of 460 V at a field current 8.7 A and which achieves rated short-circuit current at a field current of 11.2 A.

Answer (X_s = 3.21 Ω/phase = 1.29 per unit)

11- A synchronous generator with 40% synchronous reactance and negligible resistance is supplying 3/4 of the full load current at 0.7 p.f. lag and at normal terminal voltage. If the current rises to the full load value at 0.6 p.f. lag, determine the percentage change in the terminal voltage if the excitation is kept constant.

Answer (12%)

12- A 3-phase, delta-connected, 220 V, 50 Hz, 1500 rpm synchronous motor has a synchronous reactance of 4 Ω/phase. It receives an input line current of 30 A at a leading p.f. of 0.8. Find the line value of induced emf (E_f) and the load angle expressed in mechanical degrees.

Now, if the mechanical load is disconnected without change of excitation, determine the magnitude of the line current under the new condition. Neglect any losses.

Answer (E_f = 268 V, δ=24, I_L =20.8A)

13- A 3-phase, Y-connected, 4-pole, 400 V, 200 hp, synchronous motor has a synchronous reactance of 0.5Ω/phase, the resistance may be neglected. Calculate the load angle and the input current and p.f. when the machine is working at full load with its emf is adjusted to 1.0 pu based on terminal voltage.

Answer (δ=28,I_a=278.35 A, p.f.=0.776 lag)

14- A 1000 KVA, 6.6 kV, 50 Hz, 3-phase, 6-pole, Y-connected, synchronous motor has a synchronous impedance of 50 Ω/phase. The excitation is adjusted so that 50% overload is possible before it pulls out of synchronism. Calculate the necessary E_f to permit this overload margin and the output power and the input power factor.

If the output power is held constant and the motor runs at full load, calculate the input power factor and the load angle.

15- A 6 pole round rotor, 3-ph Y-connected synchronous machine has the following tests:

<u>Open circuit test:</u> 4000 V line to line at 1000 rev/min 50 A rotor current

<u>Short circuit test:</u> 300 A at 500 rev/min 50 A rotor current

Neglect the stator resistance and core losses, and assuming a linear open circuit characteristic, calculate:

(a) The machine synchronous reactance at 50 Hz,

(b) The rotor current required for the machine to operate as a motor at 0.8 power factor leading from a supply of 3.3 kV line to line with an output power of 1000 kW,

(c) The rotor current required for the machine to operate as a generator on an infinite bus of 3.3 kV line to line when delivering 1500 kVA at 0.8 power factor lagging,

(d) The load angle for (b) and (c), and Sketch the phasor diagram for (b) and (c).

Answer: 7.7 W, 69.54 A, 76 A, 24.8º, 27.4º

16- A 3-phase, 8-pole, 50 Hz, 6600 V, Y-connected synchronous motor has a synchronous impedance of 0.66+j6.5 Ω/phase. When excited to give emf 0f 4500 v/phase, it takes an input power of 2500 kW. Calculate the electromagnetic torque, input current, p.f., and the load angle.

Answer (32100 Nm, 225.7 A, 0.919 lead, 18°)

17- For a 3 ph, Y- connected 2500 kVA, 6600 V, synchronous generator operating at full load, calculate

(a) The percent voltage regulation at a power factor of 0.8 lagging,

(b) The percent voltage regulation at a power factor of 0.8 leading.

The synchronous reactance and the armature resistance are 10.4 Ω and 0.071 Ω respectively. Answer (44%, -20%)

18- A Y-connected, 3-ph, 50 Hz, 8 pole, synchronous alternator has an induced voltage of 4400 V between the lines when the rotor field current is 10 A. If this alternator is to generate 60 Hz voltage, compute the new synchronous speed and induced voltage for the same rotor current of 10 A.

Answer: E_{new}=5280 V, $n_{syn-new}$=900 rev/min

19- A 3-ph, Y-connected, 6-pole, alternator is rated at 10 kVA, 220 V, at 60 Hz. Synchronous reactance X_s=3 Ω. The no load line to neutral terminal voltage at 1000 rev/min follows the magnetization curve is as follows:

E (V)	11	38	70	102	131	156	178	193	206	215	221	224
I_f (A)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2

Determine

(a) The rated speed in rev/min,
(b) The field current required for full load operation at 0.8 power factor lagging.

Answer: nsyn=1200 rev/min, If=1.0 A

20- The following values give the open-circuit and full-load zero p.f saturation curves for a 15000-kVA. 11000 V, 3-ph, 50-Hz, star-connected turbo-alternator:

Field AT × 1000 :	10	18	24	30	40	45	50
O.C. line in (kV) :	4.9	8.4	10.1	11.5	12.8	13.3	13.65
Zero p.f.in (kV) :	—	0	—	—	—	10.2	—

Find the armature reaction, the armature reactance and the synchronous reactance. Deduce the regulation for full-load at 0.8 power lagging.

21- A 600-kVA, 3300-V, 8-pole, 3-phase, 50-Hz alternator has following characteristic :

Amp-turns/pole :	4000	5000	7000	10,000
Terminal E.M.F. :	2850	3400	3850	4400

There are 200 conductors in series per phase.

Find the short-circuit characteristic, the field ampere-turns for full-load 0.8 p.f. (lagging) and the voltage regulation.

Given that the inductive drop at full-load is 7% and that the equivalent armature reaction in amp-turns per pole at Z.P.F = 1.06 × ampere-conductors per phase per pole.

22- The following data are taken from the open circuit and short circuit characteristics of a 45 kVA, 3-ph, Y-connected, 220 V (line to line), 6-pole, 60 Hz synchronous machine:

OCC: Line to line voltage = 220 V Field current = 2.84 A

SCC: Armature current (A) 118 152 Field current (A) 2.20 2.84

Air gap line: Field current = 2.20 A Line to line voltage = 202 V

Calculate the unsaturated value of the synchronous reactance, and its saturated value at the rated voltage. Express the synchronous reactance in ohms per phase and also in per unit on the machine rating as a base.

Answer: 0.987 W per phase, 0.92 per unit, 0.836 W per phase, 0.775 per unit

23- The open-and short-circuit test readings for a 3-phase, star-connected, 1000-kVA, 2000 V, 50-Hz, synchronous generator are :

Field Current (A)	10	20	25	30	40	50
O.C. Line Voltage (V)	800	1500	1760	2000	2350	2600
S.C. Current (A)	--	200	250	300	--	--

The armature effective resistance and leakage reactance are 0.2 Ω and 0.6 Ω per phase, respectively. Draw the characteristic curves and estimate the full-load percentage regulation at 0.8 p.f. lag.